PE3101P: Decision Theory and Social Choice

Tutorial Exercises 1 — Solutions

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Instructions

These questions cover weeks 1 to 3. They are not assessed.

Throughout, you should assume that credences obey the probability axioms, unless the possibility that they do not is stated or raised.

Question 1

Recall that, for any two acts A and B over the same set of states of nature:

– A **strongly statewise dominates** B if A results in a better outcome than B in *every* state of nature.

– A **weakly statewise dominates** B if A results in an outcome at least as good as B's in *every* state of nature, and a better outcome in *at least one* state of nature.

In parts (a) to (c), give an example of the kind of decision table asked for. A small table of utilities is enough; you do not need to describe a situation. In each case, say briefly why your example has the feature asked for.

(a) A decision table with two acts and two states of nature, in which one act weakly but not strongly statewise dominates the other.

Solution. For example:

	S_1	S_2
A	2	1
B	1	1

A is at least as good as B in both states and better in S_1, so it weakly dominates B; the two are equal in S_2, so the dominance is not strong.

(b) A decision table with three acts, in which no act statewise dominates any other, either weakly or strongly — and in which no two acts have the same utility in every state.

Solution. For example:

	S_1	S_2
A	2	0
B	0	2
C	1	1

A is better than B in S_1 and worse in S_2; A is better than C in S_1 and worse in S_2; B is worse than C in S_1 and better in S_2. So in each pair neither act is at least as good as the other in every state. No two acts have the same utility in both states.

(c) A decision table with three acts A, B and C, in which A weakly but not strongly statewise dominates B, and B strongly statewise dominates C.

Solution. For example:

	S_1	S_2
A	2	1
B	1	1
C	0	0

A is better than B in S_1 and equal in S_2, so it weakly but not strongly dominates B. B is better than C in both states.

(d) State whether, in your answer to part (c), A strongly statewise dominates C, A weakly but not strongly statewise dominates C, or A does not statewise dominate C.

Solution. A **strongly** statewise dominates C: $2 > 0$ in S_1 and $1 > 0$ in S_2.

(e) Could your answer to part (d) have come out differently? If so, provide a decision table exhibiting this different answer: that is, a decision problem satisfying the conditions in part (c), but for which the answer to part (d) is different. If not, explain why.

Solution. **No.** In any table satisfying (c), A is at least as good as B in every state, and B is better than C in every state; so in every state A is better than C, which is strong dominance.

Question 2

Throughout this question, the sample space is

$$\Omega = \{1, 2, 3, 4, 5, 6\},$$

the six possible results of rolling a die.

Recall that a collection of events is **mutually exclusive** when no two of the events can obtain together — that is, when the events are pairwise disjoint — and **jointly exhaustive** when every outcome in Ω belongs to at least one of the events.

For each of the following collections of events, say whether they are

– **mutually exclusive** only;

- **jointly exhaustive** only;
- **both**; or
- **neither**.

(a) $A = \{1, 2, 3, 4\}, \quad B = \{5, 6\}$.

(b) $A = \{1, 6\}, \quad B = \{2, 5\}$.

(c) $A = \{1, 2, 3, 4\}, \quad B = \{4, 5, 6\}$.

(d) $A = \{2, 3, 4, 5\}, \quad B = \{4, 5, 6\}$.

(e) $A = \{1, 6\}, \quad B = \{2, 5\}, \quad C = \{3, 4\}$.

(f) $A = \{1, 2\}, \quad B = \{3, 4, 5\}, \quad C = \{5, 6\}$.

(g) $A = \{2, 4, 6\}, \quad B = \{1, 5\}$.

(h) $A = \{1, 2\}, \quad B = \{2, 3, 4\}, \quad C = \{4, 6\}$.

Solution.

	Overlaps	**Union**	**Answer**
(a)	none	Ω	Both
(b)	none	$\{1, 2, 5, 6\}$	Mutually exclusive only
(c)	$A \cap B = \{4\}$	Ω	Jointly exhaustive only
(d)	$A \cap B = \{4, 5\}$	$\{2, 3, 4, 5, 6\}$	Neither
(e)	none	Ω	Both
(f)	$B \cap C = \{5\}$	Ω	Jointly exhaustive only
(g)	none	$\{1, 2, 4, 5, 6\}$	Mutually exclusive only
(h)	$A \cap B = \{2\}$, $B \cap C = \{4\}$	$\{1, 2, 3, 4, 6\}$	Neither

Question 3

In each of the following, exactly one of the four claims is **guaranteed** to hold: it is true for every credence function satisfying the probability axioms, and for every pair of events A and B with positive credence. Each of the other three claims fails for some credence function. Say which claim is the guaranteed one.

Not asked for: each solution below also gives a case in which each of the other three claims holds, and one in which it fails. None of them is ruled out by the axioms, and none is guaranteed by them.

(a) (i) $\mathrm{Cr}(A \mid B) + \mathrm{Cr}(A \mid \neg B) = 1$

(ii) $\mathrm{Cr}(A \mid B) \geq \mathrm{Cr}(A)$

(iii) $\mathrm{Cr}(A \mid B) + \mathrm{Cr}(\neg A \mid B) = 1$

(iv) $\mathrm{Cr}(A \mid B) = \mathrm{Cr}(B \mid A)$

Solution. **(iii).** $A \cap B$ and $\neg A \cap B$ are disjoint and their union is B, so by Additivity $\mathrm{Cr}(A \cap B) + \mathrm{Cr}(\neg A \cap B) = \mathrm{Cr}(B)$; dividing through by $\mathrm{Cr}(B)$ gives the claim.

This also follows from the week 4 result that *conditional credences are credences*: Cr_B satisfies the probability axioms, and the claim is just $\mathrm{Cr}_B(A) + \mathrm{Cr}_B(\neg A) = 1$, the complement rule applied to Cr_B.

The others.

- **(i)** Holds when $B = A$: then $\mathrm{Cr}(A \mid B) = 1$ and $\mathrm{Cr}(A \mid \neg B) = 0$. Fails when A and B are probabilistically independent and $\mathrm{Cr}(A) \neq 1/2$: independence of A and B carries over to A and $\neg B$, so both terms are $\mathrm{Cr}(A)$ and the sum is $2\,\mathrm{Cr}(A)$.
- **(ii)** Holds when A and B are probabilistically independent: then $\mathrm{Cr}(A \mid B) = \mathrm{Cr}(A)$. Fails when $B = \neg A$ and $\mathrm{Cr}(A) > 0$: then $\mathrm{Cr}(A \mid B) = 0$, which is less than $\mathrm{Cr}(A)$.
- **(iv)** Holds when $\mathrm{Cr}(A) = \mathrm{Cr}(B)$: both sides are then $\mathrm{Cr}(A \cap B)/\,\mathrm{Cr}(A)$. Fails when $A = \Omega$ and $\mathrm{Cr}(B) < 1$: then $\mathrm{Cr}(A \mid B) = 1$ while $\mathrm{Cr}(B \mid A) = \mathrm{Cr}(B)$.

(b) (i) $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(B \mid A) \cdot \mathrm{Cr}(A)$

(ii) $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \cdot \mathrm{Cr}(B)$

(iii) $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(B \mid A) \cdot \mathrm{Cr}(B)$

(iv) $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) + \mathrm{Cr}(B) - 1$

Solution. **(i).**

$$\begin{aligned} \mathrm{Cr}(B \mid A) &= \frac{\mathrm{Cr}(A \cap B)}{\mathrm{Cr}(A)} && \text{[definition of conditional probability]} \\ \mathrm{Cr}(B \mid A) \cdot \mathrm{Cr}(A) &= \mathrm{Cr}(A \cap B) && \text{[multiplying through by } \mathrm{Cr}(A)\text{, which is positive]} \end{aligned}$$

The others.

- **(ii)** Holds when A and B are probabilistically independent — that is what independence says. Fails when A and B are mutually exclusive and $\mathrm{Cr}(A)$ and $\mathrm{Cr}(B)$ are positive: then $\mathrm{Cr}(A \cap B) = 0$ while $\mathrm{Cr}(A) \cdot \mathrm{Cr}(B) > 0$.
- **(iii)** Holds when $\mathrm{Cr}(A) = \mathrm{Cr}(B)$: by (i), the right-hand side is $\mathrm{Cr}(A \cap B) \cdot \mathrm{Cr}(B)/\,\mathrm{Cr}(A)$, which is $\mathrm{Cr}(A \cap B)$ exactly when the two credences agree. Fails when $B = \Omega$ and $\mathrm{Cr}(A) < 1$: the left-hand side is $\mathrm{Cr}(A)$ and the right-hand side is 1.
- **(iv)** Holds when A and B partition Ω: then $\mathrm{Cr}(A) + \mathrm{Cr}(B) = 1$ and $\mathrm{Cr}(A \cap B) = 0$. Fails when $\mathrm{Cr}(A) < 1/2$ and $\mathrm{Cr}(B) < 1/2$: the right-hand side is then negative, and $\mathrm{Cr}(A \cap B)$ is not.

(c) (i) $\mathrm{Cr}(A \cup B) = 1 - \mathrm{Cr}(A \cap B)$

(ii) $\mathrm{Cr}(A \cup B) \leq \mathrm{Cr}(A) + \mathrm{Cr}(B)$

(iii) $\mathrm{Cr}(A \cup B) = \mathrm{Cr}(A) + \mathrm{Cr}(B)$

(iv) $\mathrm{Cr}(A \cup B) \geq \mathrm{Cr}(A) + \mathrm{Cr}(B)$

Solution. **(ii).** $\mathrm{Cr}(A \cup B) = \mathrm{Cr}(A) + \mathrm{Cr}(B) - \mathrm{Cr}(A \cap B)$, and $\mathrm{Cr}(A \cap B) \geq 0$ by Non-negativity.

The others.

– **(i)** Holds when A and B partition Ω: then $\mathrm{Cr}(A \cup B) = 1$ and $\mathrm{Cr}(A \cap B) = 0$. Fails when $A = B$ and $\mathrm{Cr}(A) \neq 1/2$: the left-hand side is $\mathrm{Cr}(A)$ and the right-hand side is $1 - \mathrm{Cr}(A)$.

– **(iii)** Holds when A and B are mutually exclusive: that is Additivity. Fails when $\mathrm{Cr}(A \cap B) > 0$: subtracting it leaves $\mathrm{Cr}(A \cup B)$ short of $\mathrm{Cr}(A) + \mathrm{Cr}(B)$.

– **(iv)** The same two cases. Mutual exclusivity gives equality, so $\geq$ holds; and $\mathrm{Cr}(A \cap B) > 0$ makes $\mathrm{Cr}(A \cup B)$ strictly smaller than $\mathrm{Cr}(A) + \mathrm{Cr}(B)$.

Question 4

Recall the probability axioms. For any events A and B:

– **Non-negativity:** $\mathrm{Cr}(A) \geq 0$.

– **Normalisation:** $\mathrm{Cr}(\Omega) = 1$, where Ω is the sample space.

– **Additivity:** if A and B are disjoint, then $\mathrm{Cr}(A \cup B) = \mathrm{Cr}(A) + \mathrm{Cr}(B)$.

In each of the following, an agent's credences in some events are given, together with facts about the events. Say whether the credences can all be held together without violating the axioms. If they cannot, say which axiom or axioms are violated, and explain why. If one of two or more axioms must be violated, but it is not settled which one, indicate this clearly.

Recall that, for any events X and Y, the event Y is the union of the disjoint events $X \cap Y$ and $Y \setminus X$ — where $Y \setminus X$ is the event containing the outcomes that are in Y but not in X.

Where the credences below are consistent, an explanation is given. The question did not ask for one: it was enough to say that the credences are consistent with the probability axioms.

(a) $\mathrm{Cr}(P) = 0.4, \quad \mathrm{Cr}(\neg P) = 0.7.$

Solution. **Inconsistent: Additivity or Normalisation, and it is not settled which.** P and $\neg P$ are disjoint and their union is Ω, so Additivity gives $\mathrm{Cr}(\Omega) = 0.4 + 0.7 = 1.1$, while Normalisation gives $\mathrm{Cr}(\Omega) = 1$.

(b) P and Q are mutually exclusive; $\quad \mathrm{Cr}(P) = 0.3, \quad \mathrm{Cr}(Q) = 0.45, \quad \mathrm{Cr}(P \cup Q) = 0.75.$

Solution. **Consistent.** P and Q are disjoint and $0.3 + 0.45 = 0.75$, which is what Additivity requires.

(c) $P \subset Q$, so that every outcome in P is also in Q; $\quad \mathrm{Cr}(P) = 0.5, \quad \mathrm{Cr}(Q) = 0.3.$

Solution. **Inconsistent: Additivity or Non-negativity, and it is not settled which.** Since $P \subset Q$, the events P and $Q \backslash P$ are disjoint with union Q, so Additivity gives $\mathrm{Cr}(Q \backslash P) = 0.3 - 0.5 = -0.2$, which Non-negativity forbids.

(d) $\mathrm{Cr}(P) = 0.5, \quad \mathrm{Cr}(Q) = 0.5, \quad \mathrm{Cr}(P \cap Q) = 0.2, \quad \mathrm{Cr}(P \cup Q) = 0.9.$

Solution. **Inconsistent: Additivity.** P and $Q \setminus P$ are disjoint with union $P \cup Q$, so $\mathrm{Cr}(Q \setminus P) = 0.9 - 0.5 = 0.4$. But $Q \backslash P$ and $P \cap Q$ are disjoint with union Q, so $\mathrm{Cr}(Q) = 0.4 + 0.2 = 0.6$, and $\mathrm{Cr}(Q)$ is given as 0.5. Both steps use only Additivity.

Question 5

In each of the following, somebody reasons to a conclusion about the probability of something. Say whether the reasoning is correct. If it is not, say what has gone wrong.

(a) "A fair die is to be rolled. The probability that the roll is even is $1/2$, and the probability that it is at least 4 is $1/2$. These are two different ways the roll could be, so by additivity the probability that it is even, or at least 4, must be $1/2 + 1/2 = 1$."

Solution. **Incorrect.** Additivity applies only to disjoint events, and these two are not disjoint: $\{2, 4, 6\}$ and $\{4, 5, 6\}$ share the outcomes 4 and 6.

(b) "The probability that the home team wins tomorrow's match is 0.5. A team cannot win a match without scoring at least once. So the probability that the home team scores at least once is at least 0.5."

Solution. **Correct.** Winning entails scoring at least once, so the event that the home team wins is a subset of the event that it scores at least once, and a superset always has at least as great a probability as the subset.

(c) "The probability that the 8 am bus is full is 0.2, and the probability that the 9 am bus is full is 0.2. So the probability that both are full is $0.2 \times 0.2 = 0.04$."

Solution. **Incorrect.** Multiplying holds only when the two events are probabilistically independent, which nothing here establishes.

(d) "The probability that the train is delayed, given that it snows, is 0.8; and the probability that the train is delayed, given that it does not snow, is 0.15. Snow and no snow are the only two possibilities, so the probability that the train is delayed is $0.8 + 0.15 = 0.95$."

Solution. **Incorrect.** The two conditional probabilities have been added unweighted. The law of total probability weights each by the credence of the event conditioned on: $\mathrm{Cr}(\text{delayed}) = 0.8\ \mathrm{Cr}(\text{snow}) + 0.15\ \mathrm{Cr}(\neg\text{snow})$. That is a weighted average of 0.8 and 0.15, so the probability that the train is delayed lies between 0.15 and 0.8, and cannot be 0.95.

Question 6

Recall that events A and B are *mutually exclusive* when $A \cap B = \varnothing$, and that they are *probabilistically independent* when $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \cdot \mathrm{Cr}(B)$.

You may assume that $\mathrm{Cr}(\varnothing) = 0$, and that if $X \subset Y$ then $\mathrm{Cr}(X) \leq \mathrm{Cr}(Y)$.

(a) Show that if A and B are mutually exclusive, and $\mathrm{Cr}(A) > 0$ and $\mathrm{Cr}(B) > 0$, then A and B are **not** probabilistically independent.

Solution.

$$\begin{aligned} \mathrm{Cr}(A \cap B) &= \mathrm{Cr}(\varnothing) && [A \text{ and } B \text{ are mutually exclusive}] \\ &= 0 && [\text{given}] \end{aligned}$$

But $\mathrm{Cr}(A) \cdot \mathrm{Cr}(B) > 0$, since both factors are positive. So $\mathrm{Cr}(A \cap B) \neq \mathrm{Cr}(A) \cdot \mathrm{Cr}(B)$.

(b) Show that if $\mathrm{Cr}(A) = 0$, then A and B are probabilistically independent.

Solution. We want to show that $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \cdot \mathrm{Cr}(B)$. We show that both sides are equal to 0.

Right-hand side:

$$\mathrm{Cr}(A) \cdot \mathrm{Cr}(B) = 0 \cdot \mathrm{Cr}(B) = 0 \qquad [\text{assumption}]$$

Left-hand side:

$$\begin{aligned} \mathrm{Cr}(A \cap B) &\leq \mathrm{Cr}(A) && [\text{given, since } A \cap B \subset A] \\ \mathrm{Cr}(A) &= 0 && [\text{assumption}] \\ \mathrm{Cr}(A \cap B) &\leq 0 && [\text{from the previous two lines}] \\ \mathrm{Cr}(A \cap B) &\geq 0 && [\text{Non-negativity}] \\ \mathrm{Cr}(A \cap B) &= 0 && [\text{from the previous two lines}] \end{aligned}$$

Question 7

Recall two ways of saying that events A and B are probabilistically independent. The first way is:

$$\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \cdot \mathrm{Cr}(B).$$

The second says that learning B makes no difference to how likely A is:

$$\mathrm{Cr}(A \mid B) = \mathrm{Cr}(A).$$

Recall also the definition of conditional probability:

$$\mathrm{Cr}(A \mid B) = \frac{\mathrm{Cr}(A \cap B)}{\mathrm{Cr}(B)}.$$

Assume throughout that $\mathrm{Cr}(B) > 0$.

(a) Show that if $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \cdot \mathrm{Cr}(B)$, then $\mathrm{Cr}(A \mid B) = \mathrm{Cr}(A)$.

Solution.

$$\begin{aligned}
\mathrm{Cr}(A \mid B) &= \frac{\mathrm{Cr}(A \cap B)}{\mathrm{Cr}(B)} && \text{[definition of conditional probability]} \\
&= \frac{\mathrm{Cr}(A) \cdot \mathrm{Cr}(B)}{\mathrm{Cr}(B)} && \text{[assumption]} \\
&= \mathrm{Cr}(A) && \text{[since } \mathrm{Cr}(B) > 0\text{]}
\end{aligned}$$

(b) Show the converse: if $\mathrm{Cr}(A \mid B) = \mathrm{Cr}(A)$, then $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \cdot \mathrm{Cr}(B)$.

Solution.

$$\begin{aligned}
\mathrm{Cr}(A \mid B) &= \frac{\mathrm{Cr}(A \cap B)}{\mathrm{Cr}(B)} && \text{[definition of conditional probability]} \\
\mathrm{Cr}(A \mid B) \cdot \mathrm{Cr}(B) &= \mathrm{Cr}(A \cap B) && \text{[multiplying both sides by } \mathrm{Cr}(B)\text{]} \\
\mathrm{Cr}(A) \cdot \mathrm{Cr}(B) &= \mathrm{Cr}(A \cap B) && \text{[assumption, } \mathrm{Cr}(A \mid B) = \mathrm{Cr}(A)\text{]}
\end{aligned}$$

Question 8

A fair six-sided die is to be rolled, so that the sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$ and you assign each outcome credence $1/6$. Consider the following four events.

$$A = \{1, 3, 5\}, \qquad B = \{3, 4, 5, 6\}, \qquad C = \{5, 6\}, \qquad D = \{2\}.$$

Identify **every** pair of these events that is probabilistically independent, and show your working in each case — including for the pairs that are not.

Solution. The credences are $\mathrm{Cr}(A) = 1/2$, $\mathrm{Cr}(B) = 2/3$, $\mathrm{Cr}(C) = 1/3$ and $\mathrm{Cr}(D) = 1/6$.

Pair	**Intersection**	$\mathrm{Cr}(X \cap Y)$	$\mathrm{Cr}(X) \cdot \mathrm{Cr}(Y)$	
A, B	$\{3, 5\}$	$1/3$	$1/2 \cdot 2/3 = 1/3$	Independent
A, C	$\{5\}$	$1/6$	$1/2 \cdot 1/3 = 1/6$	Independent
A, D	$\varnothing$	0	$1/2 \cdot 1/6 = 1/12$	Not
B, C	$\{5, 6\}$	$1/3$	$2/3 \cdot 1/3 = 2/9$	Not
B, D	$\varnothing$	0	$2/3 \cdot 1/6 = 1/9$	Not
C, D	$\varnothing$	0	$1/3 \cdot 1/6 = 1/18$	Not

The independent pairs are A, B and A, C.

Question 9

Recall the betting interpretation of credences. Write an *\$S bet on A* for a deal that pays $\$S$ if A is true and nothing otherwise.

The betting interpretation of credences. An agent whose credence in A is p will buy an $\$S$ bet on A for any price below $\$pS$, and will sell one for any price above $\$pS$.

Buying a bet means paying money in order to receive the uncertain payout; selling a bet means receiving money in return for taking on the risk of the uncertain loss. Prices are paid when a bet is agreed; the payout is settled afterwards, once it is known whether the event obtained.

Assume the betting interpretation throughout. In each of the following, the agent's credences violate one of the probability axioms. In each case:

- say which axiom her credences violate;
- state a single bet, **with its price**, that the agent will accept according to the betting interpretation; and
- show that, having accepted it, she loses money *whatever happens*.

Take the stake to be $S = 100$ throughout. A negative price means that the money flows the other way: an agent who *sells* a bet at a price of $-\$20$ pays \$20 to the buyer, and still owes the $\$S$ payout if the event obtains.

(a) $\mathrm{Cr}(A) = -0.4$, for some event A.

Solution. **Non-negativity is violated.** Her fair price for a \$100 bet on A is $-0.4 \times 100 = -\$40$, so she will sell one at any price above $-\$40$: say she sells at $-\$20$, paying \$20 for the privilege and still owing \$100 if A obtains.

	A obtains	A does not
Received for the bet	$-\$20$	$-\$20$
Payout owed	$-\$100$	$\$0$
Net	$-\$120$	$-\$20$

The price has to be negative: at \$0 or more she would lose nothing when A fails.

(b) $\mathrm{Cr}(\Omega) = 0.9$, where Ω is the sample space.

Solution. **Normalisation is violated.** Her fair price for a \$100 bet on Ω is \$90, so she will sell one at any price above \$90: say she sells at \$91. Ω obtains whatever happens, so she receives \$91 and pays out \$100 — a certain loss of \$9.

(c) $\mathrm{Cr}(\Omega) = 1.2$.

Solution. **Normalisation is violated.** Her fair price for a \$100 bet on Ω is \$120, so she will buy one at any price below \$120: say she buys at \$119. Ω obtains whatever happens, so she pays \$119 and receives \$100 — a certain loss of \$19.

Question 10

Recall that, in a decision problem, a **dependency hypothesis** lists, for each act available, which outcome would result were that act performed.

It will help to have a compact notation: write $\langle o, o' \rangle$ for the dependency hypothesis on which the first act listed would result in outcome o, and the second act in outcome o'.

Each of the following describes a decision situation in words. The outcomes have been given letters, for your convenience in writing the dependency hypotheses. In each case, first identify the acts available and the possible outcomes; then list **every** dependency hypothesis.

(a) You are deciding whether to take the bus to campus or to walk. Whichever you do, either you will arrive on time (a), or you will be late (b).

Solution. **Acts:** take the bus; walk. **Outcomes:** arrive on time (a); be late (b).

$$\langle a, a\rangle, \qquad \langle a, b\rangle, \qquad \langle b, a\rangle, \qquad \langle b, b\rangle.$$

(b) You have a headache, and you are deciding whether to take a painkiller. Either the headache will be (g)one within the hour, or it will (n)ot be gone.

Solution. **Acts:** take the painkiller; do not take it. **Outcomes:** the headache is gone within the hour (g); it is not (n).

$$\langle g, g\rangle, \qquad \langle g, n\rangle, \qquad \langle n, g\rangle, \qquad \langle n, n\rangle.$$

(c) You have one seedling, and you are deciding whether to plant it in the garden bed or to keep it in a pot on the balcony. Either it will (f)lower this year, or it will (n)ot.

Solution. **Acts:** plant it in the garden bed; keep it in a pot. **Outcomes:** it flowers this year (f); it does not (n).

$$\langle f, f\rangle, \qquad \langle f, n\rangle, \qquad \langle n, f\rangle, \qquad \langle n, n\rangle.$$

Question 11

An agent faces a choice between two acts, A_1 and A_2. There are two states of nature, S_1 and S_2, and **these states are causally independent of the acts**. Her utilities are as follows.

	S_1	S_2
A_1	20	0
A_2	22	2

Her credences are

$$\mathrm{Cr}(S_1 \mid A_1) = 0.8, \qquad \mathrm{Cr}(S_1 \mid A_2) = 0.2,$$
$$\mathrm{Cr}(A_1) = 0.75, \qquad \mathrm{Cr}(A_2) = 0.25.$$

She is certain that she will perform exactly one of the two acts.

(a) Using the law of total probability, find $\mathrm{Cr}(S_1)$ and $\mathrm{Cr}(S_2)$.

Solution. A_1 and A_2 are mutually exclusive and jointly exhaustive, so

$$\begin{aligned}\mathrm{Cr}(S_1) &= \mathrm{Cr}(A_1)\,\mathrm{Cr}(S_1 \mid A_1) + \mathrm{Cr}(A_2)\,\mathrm{Cr}(S_1 \mid A_2) \\ &= (0.75)(0.8) + (0.25)(0.2) = 0.6 + 0.05 = 0.65,\end{aligned}$$

and S_1 and S_2 are complements, so $\mathrm{Cr}(S_2) = 1 - 0.65 = 0.35$.

(b) Find $\mathrm{Cr}(S_2 \mid A_1)$ and $\mathrm{Cr}(S_2 \mid A_2)$. (Note that, as S_1 and S_2 are states of nature, they are mutually exclusive and jointly exhaustive.)

Solution. By the week 4 result that *conditional credences are credences*, Cr_{A_1} and Cr_{A_2} satisfy the probability axioms, so the complement rule applies to each. As S_1 and S_2 are complements,

$$\mathrm{Cr}(S_2 \mid A_1) = 1 - \mathrm{Cr}(S_1 \mid A_1) = 0.2, \qquad \mathrm{Cr}(S_2 \mid A_2) = 1 - \mathrm{Cr}(S_1 \mid A_2) = 0.8.$$

(c) Taking the states of nature to be S_1 and S_2 as given above, does either act statewise dominate the other? If so, which, and is the dominance strong or weak?

Solution. A_2 **strongly** statewise dominates A_1: $22 > 20$ in S_1 and $2 > 0$ in S_2.

(d) The *causal* expected utility of an act A, where the states are causally independent of the acts, is

$$\mathrm{EU}_c(A) = \sum_i \mathrm{Cr}(S_i)\, U(A, S_i).$$

Calculate $\mathrm{EU}_c(A_1)$ and $\mathrm{EU}_c(A_2)$. Which act does the causal theory recommend?

Solution.

$$\begin{aligned}
\mathrm{EU}_c(A_1) &= (0.65)(20) + (0.35)(0) = 13,\\
\mathrm{EU}_c(A_2) &= (0.65)(22) + (0.35)(2) = 14.3 + 0.7 = 15.
\end{aligned}$$

The causal theory recommends A_2.

(e) The *evidential* expected utility of an act A is

$$\mathrm{EU}_e(A) = \sum_i \mathrm{Cr}(S_i \mid A)\, U(A, S_i).$$

Calculate $\mathrm{EU}_e(A_1)$ and $\mathrm{EU}_e(A_2)$. Which act does the evidential theory recommend?

Solution.

$$\begin{aligned}
\mathrm{EU}_e(A_1) &= (0.8)(20) + (0.2)(0) = 16,\\
\mathrm{EU}_e(A_2) &= (0.2)(22) + (0.8)(2) = 4.4 + 1.6 = 6.
\end{aligned}$$

The evidential theory recommends A_1.