

Optional Exercises (Week 5)

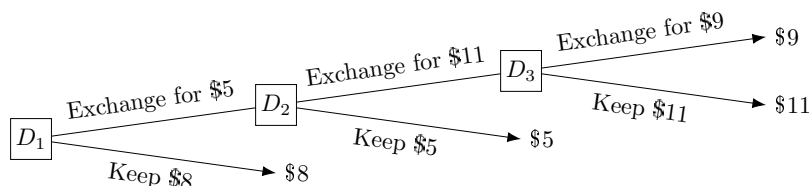
- **Myopic choice:** compare the holdings or prospects received immediately, ignoring later choices.
- **Naive choice:** select a continuation using current preferences, without anticipating later departures from it. At each later decision node, choose afresh.
- **Sophisticated choice:** work backwards, predicting later choices and comparing their consequences using the preferences held at the current decision node.
- **Resolute choice:** select a complete plan using the initial preferences and carry it out even if it later goes against what the agent's preferences would have been without the plan.

A complete plan specifies a choice at every decision node, including nodes it does not reach. The whole tree is known in advance. Squares are decision nodes; circles are chance nodes.

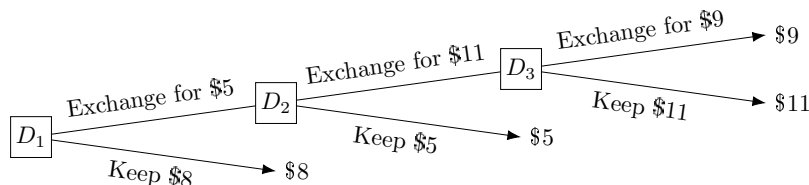
Question 1

An agent starts with \$8 and always prefers more money to less. Each exchange replaces the current holding by the stated amount. Keeping the current holding ends the sequence.

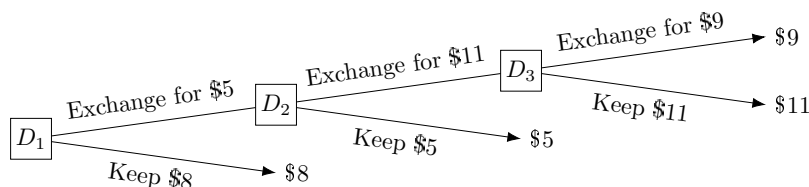
Myopic choice



Naive choice



Sophisticated choice

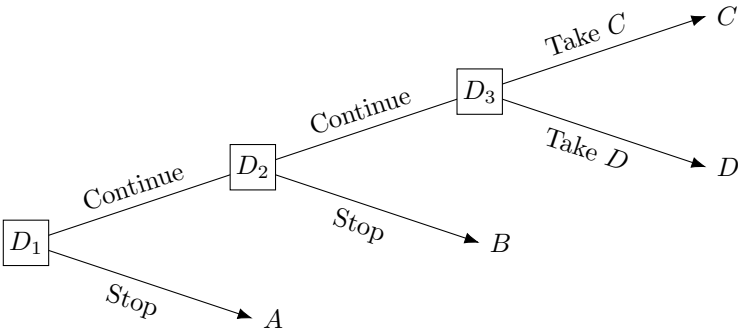


- (a) On the first tree, shade the chosen branches or state the choice at every decision node, including any that will not be reached. State the final holding.
- (b) Do the same for naive choice on the second tree.
- (c) Do the same for sophisticated choice on the third tree.
- (d) At D_1 , which two holdings does the myopic agent compare? Which two final holdings does the naive agent compare? Which two final holdings does the sophisticated agent compare?

Question 2

The agent’s utilities for the four possible final outcomes are:

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
At D_1	6	3	12	8
At D_2	0	10	1	7
At D_3	0	10	1	7



- (a) Complete the table. Give the choices at D_2 and D_3 even when an earlier choice means they will not be reached.

Procedure	At D_1	At D_2	At D_3	Final outcome
Naive				
Sophisticated				
Resolute				

- (b) For naive choice, write down the outcome anticipated at D_1 and the outcome actually obtained.
- (c) At D_2 , which final outcome will the resolute agent obtain by carrying out the initial plan? If instead they started at D_2 without a pre-existing plan, which final outcome would they plan to reach?

Question 3

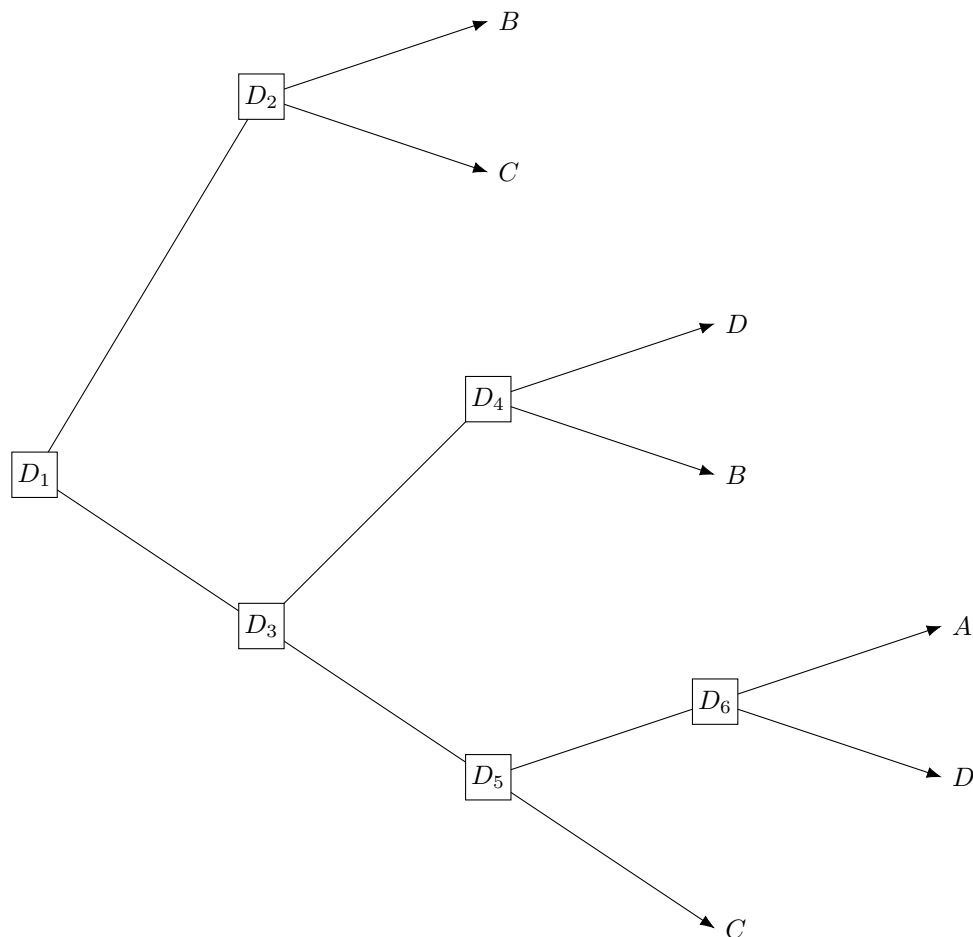
The agent's strict preferences remain the same throughout:

$$A \succ B, \quad B \succ C, \quad C \succ D, \quad D \succ A,$$

and, for the remaining pairs, $C \succ A$ and $D \succ B$. From a pair of outcomes, the agent chooses the strictly preferred one. When more than two distinct outcomes are anticipated, the agent selects A if it is among them, and otherwise selects B . These rules apply in both parts. A sophisticated agent does not anticipate the results of a later choice that they predict they will not make.

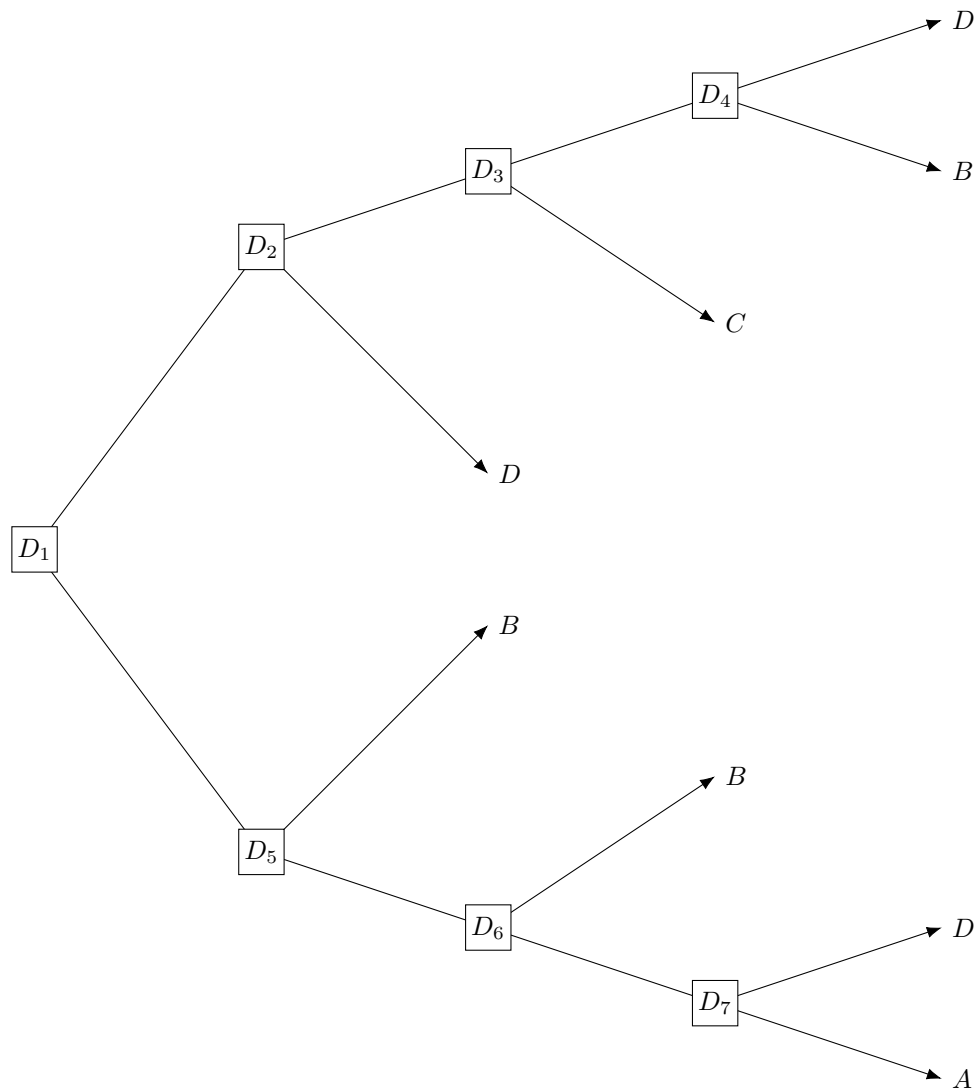
In each tree, label one branch at every decision node with each of N , S and R , for naive, sophisticated and resolute choice. A branch may carry several letters. For R , label the branches along the unique route to the outcome selected at D_1 . Complete the R labels at unreached decision nodes in any way; these labels do not affect the route or final outcome. State the final outcome for each procedure.

(a)



Question 3 (continued)

(b) Use the same preferences and selection rules as in part (a).

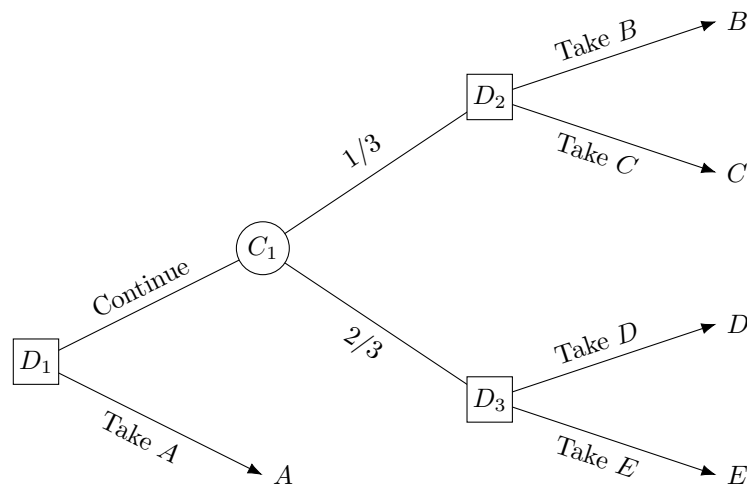


Question 4

Write plans here as $(D_1 : \dots, D_2 : \dots, D_3 : \dots)$.

The probabilities on the chance branches are the agent's credences before the chance event is resolved. The agent evaluates prospects by expected utility. Utilities at the decision times are:

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
At D_1	4	12	3	6	9
At D_2	0	1	8	10	2
At D_3	0	1	8	10	2



- Find a complete plan that maximises expected utility using the utilities as evaluated at D_1 . Calculate its expected utility.
- For resolute choice, either mark the selected branches with **R**, or state the plan's choices in writing, including choices at decision nodes that will not be reached. Give the probability of each final outcome under the decision selected by the resolute agent at D_1 .
- For sophisticated choice, either mark the chosen branches with **S**, or state the choices in writing, including choices at decision nodes that will not be reached. Calculate the initial expected utility of Continue under this prediction and give the resulting probability of each final outcome. A branch may carry both letters.

Question 5

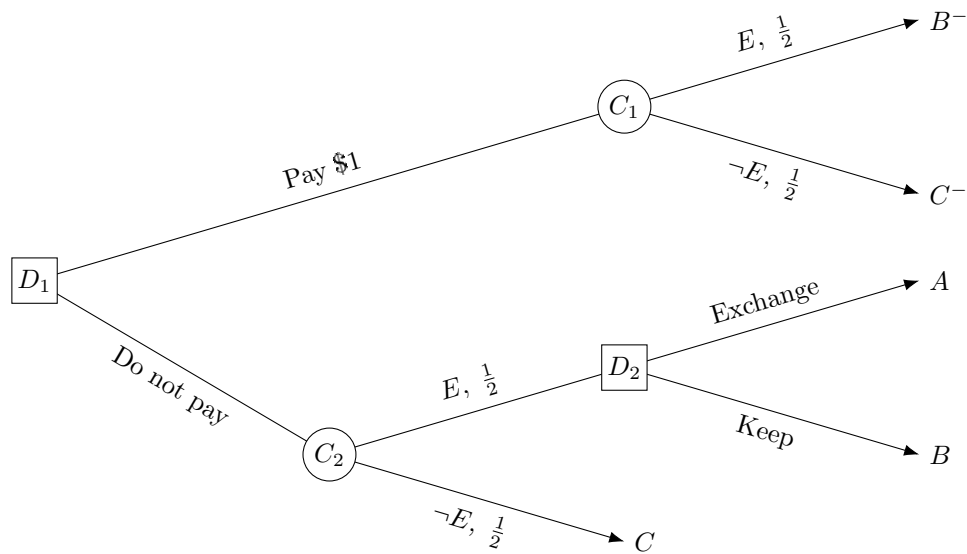
An agent initially holds a lottery $B[\frac{1}{2}]C$: they will receive B if an event E occurs and C otherwise. The agent's initial credence in E is $\frac{1}{2}$, as is its probability.

Before learning whether E occurs, the agent can pay \$1 to end all further offers. If they do not pay and E occurs, they will be offered a free exchange of B for A . If E does not occur, there is no further choice.

Write B^- and C^- for receiving B and C , respectively, with \$1 deducted. The agent's preferences are

$$\begin{aligned} A &\succ B, & B[\tfrac{1}{2}]C &\succ A[\tfrac{1}{2}]C, \\ B[\tfrac{1}{2}]C &\succ B^-[\tfrac{1}{2}]C^-, & B^-[\tfrac{1}{2}]C^- &\succ A[\tfrac{1}{2}]C. \end{aligned}$$

These preferences do not change. Both chance nodes below concern the same event E .



- Given $A \succ B$, what preference between $A[\frac{1}{2}]C$ and $B[\frac{1}{2}]C$ would Independence require? Do the stated preferences satisfy this requirement?
- Suppose the agent follows sophisticated choice. What will they choose at D_2 if it is reached?
- Using your answer to (b), write down the lottery the sophisticated agent anticipates receiving if they do not pay at D_1 . Which branch will they choose at D_1 , and which lottery will they receive?
- Specify the choices at D_1 and D_2 that would preserve the original lottery $B[\frac{1}{2}]C$. Compare the outcome under this plan with the sophisticated agent's outcome if E occurs, and then if E does not occur. How much money does the sophisticated agent lose in each case relative to this plan?

Question 6

State whether each statement is **true or false**. No proofs or explanations are required.

By an agent being *vulnerable to money pumps*, we mean that there exists a decision tree in which a cost-free plan would preserve the agent's initial holding, but following their preferences and dynamic decision rule leads to a guaranteed loss with no change in what they otherwise hold. The claims concern the money-pump arguments covered in the lecture.

- (i) Any agent who follows naive choice is vulnerable to money pumps.
- (ii) An agent can use resolute choice to avoid a money pump, even if her preferences are cyclic.
- (iii) An agent who follows sophisticated choice is not vulnerable to money pumps, even if her preferences are cyclic.
- (iv) If an agent's preferences satisfy acyclicity, they satisfy transitivity.
- (v) An agent applies resolute choice by paying a small cost to prevent herself from having the ability to choose certain options at a later decision node.
- (vi) Odysseus applied resolute choice in order to escape the sirens while still listening to their song.
Clarificatory note: in answering this question, you should imagine that Homer's epic poem, The Odyssey, is a factual account of real events.
- (vii) Employing naive choice can block certain kinds of money pumps.
- (viii) Money pump arguments are intended to establish that certain patterns of preferences are irrational.
- (ix) Money pump arguments are intended to establish that following myopic choice is irrational.
- (x) An agent can avoid all of the money pumps we have seen if they accept all of the axioms of Expected Utility Theory.