Decision and Social Choice

PE3101P — Lecture 5: Money pumps and dynamic choice

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Today

The Addition Rule

Proposition

If Cr satisfies the probability axioms, then

$$\mathrm{Cr}(A \cup B) = \mathrm{Cr}(A) + \mathrm{Cr}(B) - \mathrm{Cr}(A \cap B).$$

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The overlap is counted twice when we add $\mathrm{Cr}(A)$ and $\mathrm{Cr}(B)$.

Proving the Addition Rule

$$\mathrm{Cr}(A \cup B)$$

What we want to calculate.

Proving the Addition Rule

$$\mathrm{Cr}(A \cup B) = \mathrm{Cr}\big(\boldsymbol{A} \cup \boldsymbol{(B \setminus A)}\big)$$

Split the union into A and the part of B outside A.

Proving the Addition Rule

$$\mathrm{Cr}(A \cup B) = \boldsymbol{\mathrm{Cr}(A) + \mathrm{Cr}(B \setminus A)}$$

Finite additivity: those two sets are disjoint.

Proving the Addition Rule

$$\mathrm{Cr}(B) = \mathrm{Cr}\Big(\boldsymbol{(B \setminus A)} \cup \boldsymbol{(A \cap B)}\Big)$$

Now split B into its parts outside and inside A.

Proving the Addition Rule

$$\mathrm{Cr}(B) = \mathbf{Cr}(\boldsymbol{B} \setminus \boldsymbol{A}) + \mathbf{Cr}(\boldsymbol{A} \cap \boldsymbol{B})$$

Finite additivity again.

Proving the Addition Rule

$$\mathbf{Cr}(B \setminus A) = \mathbf{Cr}(B) - \mathbf{Cr}(A \cap B)$$

Subtract $\mathrm{Cr}(A \cap B)$ from both sides.

Proving the Addition Rule

$$\mathrm{Cr}(A \cup B) = \mathrm{Cr}(A) + \boldsymbol{\mathrm{Cr}(B) - \mathrm{Cr}(A \cap B)}$$

Substitute for $\mathrm{Cr}(B \setminus A)$ in the first calculation.

Prospects and Preferences

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- A prospect specifies the probabilities of its outcomes. Each prospect has only finitely many possible outcomes.
- $P \succsim Q$: P is at least as good as Q, according to your preferences. $P \succ Q$: strictly preferred. $P \sim Q$: indifferent.
- Stochastic equivalence lets us identify gambles with the same outcome probabilities.

The Expected Utility Theorem

The Expected Utility Theorem

Your preferences over prospects satisfy Completeness, Transitivity, Independence and Continuity if and only if some function u on outcomes represents them by expected utility:

$$P \succsim Q \quad \text{iff} \quad \sum_x P(x)u(x) \geq \sum_x Q(x)u(x).$$

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- The ranking by expected utility is exactly your preference ranking.
- The units and zero are arbitrary: $u' = au + c$, where $a > 0$.

Completeness and Transitivity

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For any prospects A and B, $A \succsim B$ or $B \succsim A$.

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- Completeness rules out gaps.
- Transitivity makes the comparisons fit together.

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	probability p	probability $1 - p$
ApC	A	C
BpC	B	C

The common chance of C does not reverse the ranking of A and B.

Independence

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For any prospects A, B, C and any $p \in (0,1)$,

$$A \succsim B \quad \text{iff} \quad ApC \succsim BpC.$$

	Democrat wins, 0.6	Republican wins, 0.4
Prospect 1	\$50	a coffee
Prospect 2	a cinema ticket	a coffee

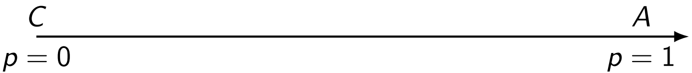
Prefer \$50 to a cinema ticket? Then prefer Prospect 1. The coffee comes either way, so it drops out.

Continuity

Continuity

If $A \succ B \succ C$, there is some $p \in (0, 1)$ such that

$$B \sim ApC.$$



B sits strictly between C and A. Slide p from 0 to 1: somewhere the mixture must match B exactly.

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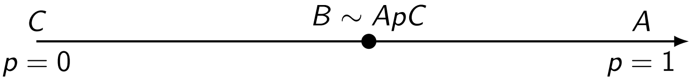
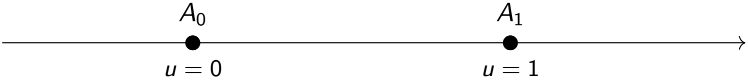


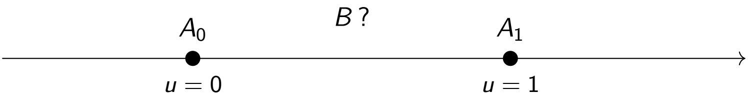
Here $p = 0.55$: a 0.55 chance of A, otherwise C, is exactly as good as B.

The Idea of the Proof



Pick any two prospects with $A_1 \succ A_0$, and give them utilities 1 and 0.
Any pair will do: the zero and the unit are ours to choose.

The Idea of the Proof



Now take any third prospect B. Where does it go? By Completeness and Transitivity there are exactly five cases.

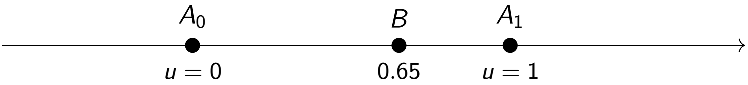
The Idea of the Proof

Case 1: $B \sim A_1$. Then $u(B) = 1$.

The Idea of the Proof

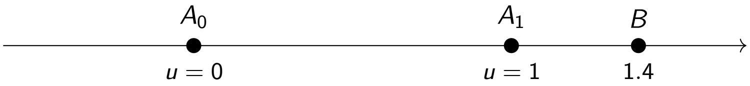
Case 2: $B \sim A_0$. Then $u(B) = 0$.

The Idea of the Proof



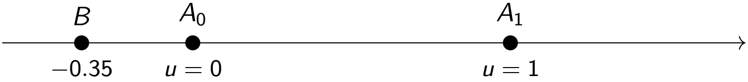
Case 3: $A_1 \succ B \succ A_0$. Continuity gives a p with $B \sim A_1 \, p \, A_0$. Set $u(B) = p$; here $p = 0.65$. A better B needs a bigger chance of A_1, so it sits further right.

The Idea of the Proof



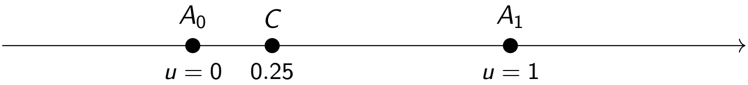
Case 4: $B \succ A_1$. Now A_1 is the middle one, so $A_1 \sim B\,q\,A_0$ for some q.
If u respects mixing, $1 = q\,u(B)$, so $u(B) = 1/q > 1$. Here $q \approx 0.71$.

The Idea of the Proof



Case 5: $A_0 \succ B$. Now A_0 is the middle one: $A_0 \sim A_1 \, r \, B$. Respecting mixing, $0 = r + (1 - r)\, u(B)$, so $u(B) = -r/(1 - r) < 0$. Here $r \approx 0.26$.

The Idea of the Proof



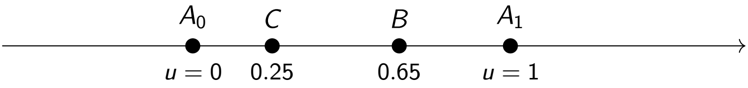
We have assigned utilities. Now: do they respect mixtures?

Introduce another prospect C between the anchors, with $u(C) = 0.25$:

$$C \sim A_1 \, 0.25 \, A_0.$$

These are equally preferred, not identical prospects.

The Idea of the Proof

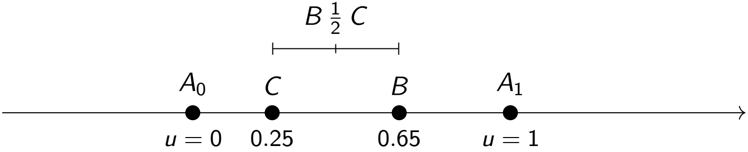


Take B between the anchors too, with $u(B) = 0.65$:

$$B \sim A_1 \, 0.65 \, A_0.$$

Each prospect is indifferent to its own lottery over A_0 and A_1.

The Idea of the Proof

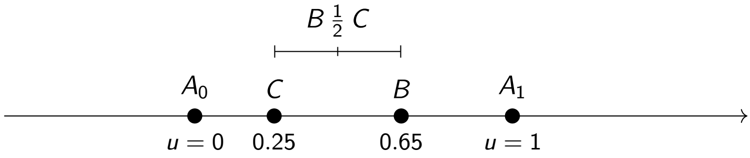


What happens when we mix B and C equally?

$$B \frac{1}{2} C$$

We want to show that its utility is $\frac{1}{2}u(B) + \frac{1}{2}u(C)$.

The Idea of the Proof

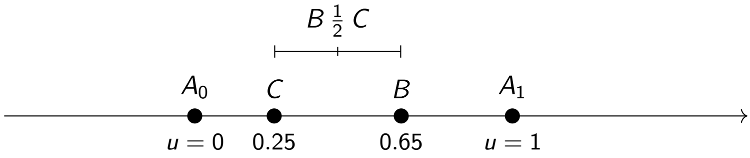


First replace B by its equally preferred anchor lottery.

Independence preserves indifference inside the mixture:

$$B\,\frac{1}{2}\,C \sim \left(A_1\,0.65\,A_0\right)\frac{1}{2}\,C.$$

The Idea of the Proof

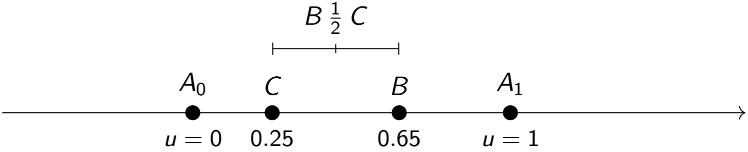


Now replace C in the same way. Again, Independence preserves indifference:

$$B\,\frac{1}{2}\,C \sim \left(A_1\,0.65\,A_0\right)\frac{1}{2}\left(A_1\,0.25\,A_0\right).$$

On the right, either branch leads to a lottery over the same two anchors.

The Idea of the Proof



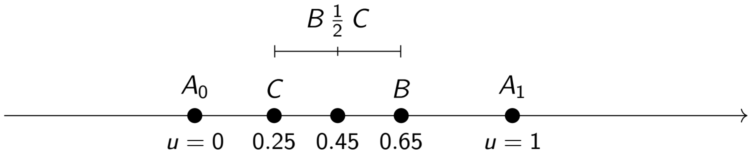
Reduce that compound lottery. Its total probability of A_1 is

$$\frac{1}{2}(0.65) + \frac{1}{2}(0.25) = 0.45.$$

Otherwise it gives A_0. By stochastic equivalence, it is indifferent to

$$A_1 \, 0.45 \, A_0.$$

The Idea of the Proof



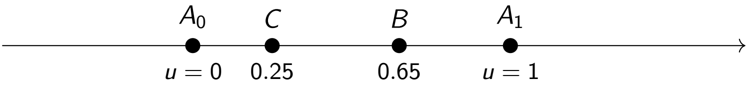
So, by Transitivity,

$$B \, \frac{1}{2} \, C \sim A_1 \, 0.45 \, A_0.$$

Our utility assignment therefore gives

$$u(B \, \frac{1}{2} \, C) = 0.45 = \tfrac{1}{2}u(B) + \tfrac{1}{2}u(C).$$

The Idea of the Proof

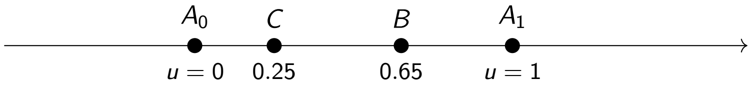


The same reasoning works for any two prospects between the anchors and any mixing probability p:

$$B\, p\, C \sim A_1 \left(pu(B) + (1-p)u(C) \right) A_0 .$$

Hence $u(B\, p\, C) = pu(B) + (1-p)u(C)$.

The Idea of the Proof



And the order comes out right: $B \succ C$ exactly when B matches a bigger chance of A_1 than C does, because a bigger chance of the better anchor is better. So u ranks prospects just as $\succsim$ does.

What the Theorem Gives Us

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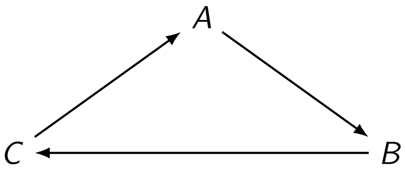
- It does not, by itself, prove that rationality requires those constraints.
- Today: can money-pump arguments justify ruling out preferences which violate some of the axioms?

Cyclic Preferences

$$A \succ B, \qquad B \succ C, \qquad C \succ A.$$



An arrow runs from the preferred option to the less preferred one.

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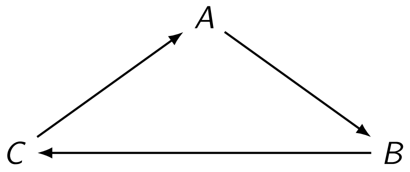

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These comparisons cannot be represented by real-valued utility.

The Money-Pump Idea

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The Money-Pump Idea

- An exploiter offers a sequence of trades. Each trade can look attractive when it is offered.
- You finish with the same thing you started with, but less money.
- As with a Dutch book, the proposed test is vulnerability to a predictable loss.
- The target is the structure of your preferences.

These arguments closely follow Gustafsson, *Money-Pump Arguments* (2022).

Strict Preference Survives Small Losses

$$A \succ B$$

$$B \succ C$$

$$C \succ A$$

The cycle: three claims, each on its own. We are not assuming Transitivity, so nothing else follows from them.

Strict Preference Survives Small Losses

$$A \succ B$$

$$B \succ C$$

$$C \succ A$$

$$A \succ A^-$$

$$A^- \succ B$$

Let A^- be A with a small amount of money removed. Then $A \succ A^-$; and since $A \succ B$, a small enough cost keeps $A^- \succ B$.

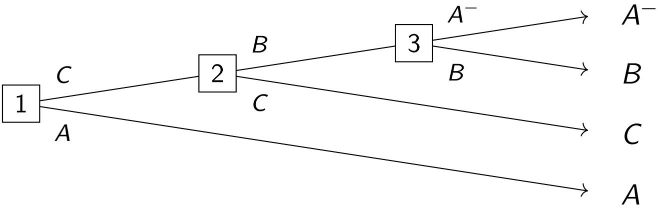
Strict Preference Survives Small Losses

$$A \succ B$$
$$B \succ C$$
$$C \succ A$$
$$A \succ A^-$$
$$A^- \succ B$$
$$C \succ A^-$$

And since $C \succ A$: if A^- is only a tiny bit worse than A, then $C \succ A^-$ too. Assumptions about small costs, not consequences of the cycle.

The Standard Money Pump

Start with A. Refusing an offer ends the sequence.

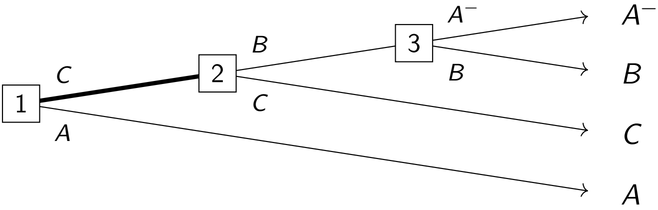


$$C \succ A, \quad B \succ C, \quad A^{-} \succ B.$$

Gustafsson (2022: 7), Figure 1.

The Standard Money Pump

Myopic choice. Node 1: $C \succ A$. Accept.

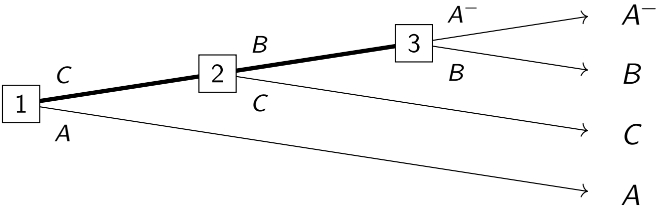


$$C \succ A, \quad B \succ C, \quad A^- \succ B.$$

Gustafsson (2022: 7), Figure 1.

The Standard Money Pump

Node 2: $B \succ C$. Accept.



$$C \succ A, \quad B \succ C, \quad A^- \succ B.$$

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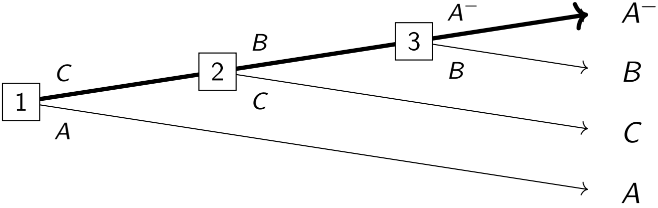
The Standard Money Pump

Node 3: $A^{-} \succ B$. Pay. You started with A and end with A^{-}.



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Myopic and Naive Choice

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At every node, choose whatever you would most prefer to have right away, regardless of what the rest of the decision tree looks like.

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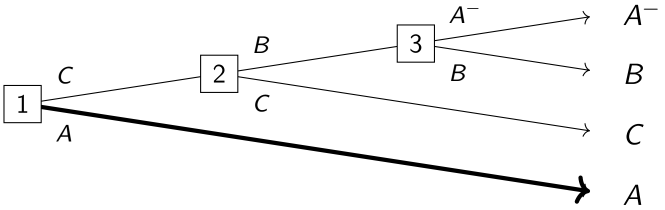
Naive choice

Survey the outcomes still available, and move towards the one you most prefer, without taking into account what you might do at future decision nodes.

Myopic and Naive Choice

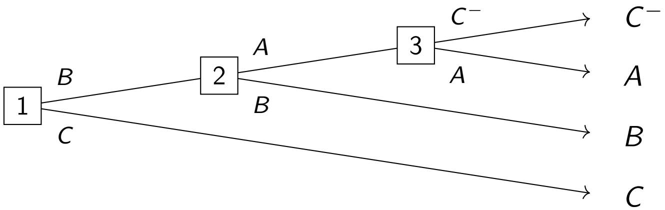
Naive choice in the standard pump. Suppose that, when A, B and C are all reachable, you want A.



At node 1, A is reachable by refusing. So refuse, and keep A. No pump.

A Pump for Naive Choice

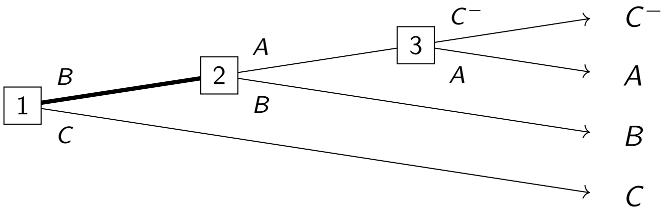
Turn the offers around. Start with C; again you want A whenever all three are reachable.



$B \succ C, \quad A \succ B, \quad C^- \succ A$ (since $C \succ A$ and C^- is only a little worse than C).

A Pump for Naive Choice

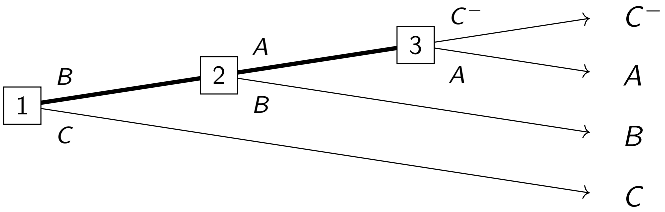
Node 1: A is reachable only by trading. Head for it: accept B.



$B \succ C$, $\quad A \succ B$, $\quad C^- \succ A$ (since $C \succ A$ and C^- is only a little worse than C).

A Pump for Naive Choice

Node 2: still heading for A: accept.



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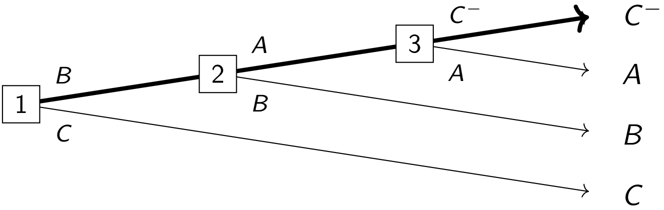
A Pump for Naive Choice

Node 3: only A and C^- are left, and $C^- \succ A$. Accept. You started with C and end with C^-.



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- So the argument assumes that you **know the whole decision tree in advance**: no surprises.
- And that your **preferences stay fixed** throughout: the cycle is not a change of mind.
- Then whatever you lose, you lose because of the preferences themselves.

Sophisticated Choice

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Choose at each node by working backwards: predict what you would choose at later nodes, and assess your current options in light of those predictions.

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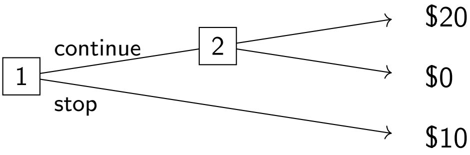
Choose at each node by working backwards: predict what you would choose at later nodes, and assess your current options in light of those predictions.

- In a finite tree, start at the last choices and work backwards.

Adapted from Gustafsson (2022: 8).

A Simple Example

More money is better. But you know that, if you reach node 2, you will mistakenly choose $0.



What should you do at node 1?

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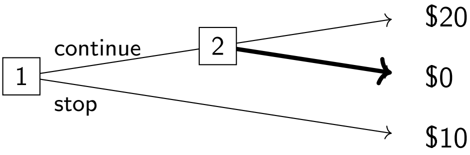

Predict your actual choice at node 2: \$0, even though \$20 is available.

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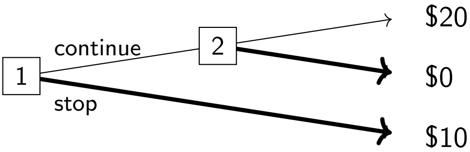

Sophisticated choice: stop for \$10, rather than continue to \$0.
Naive choice: continue towards the reachable \$20.

Decision-Tree Separability

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What it is rational to choose at a node depends only on the part of the tree still reachable from that node.

Decision-Tree Separability

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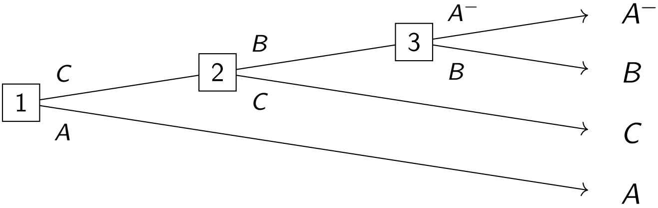
- At the final node of the standard pump, compare B and A^-.
- Having previously been able to keep A does not alter that comparison.
- We will return to this assumption when we consider resolute choice.

Foresight in the Standard Pump

$A \succ A^{-} \succ B \succ C \succ A$, and $C \succ A^{-}$.



Work backwards from node 3.

Foresight in the Standard Pump

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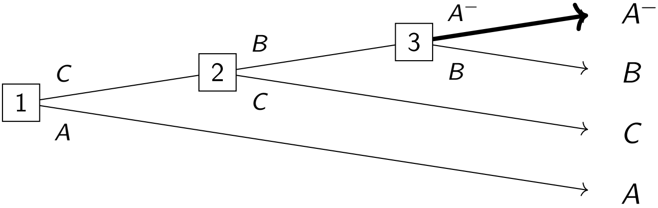

Node 3: pay, because $A^- \succ B$.

Foresight in the Standard Pump

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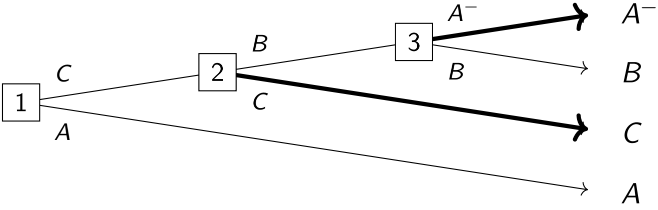

Node 2: accepting leads to A^-. Refuse, because $C \succ A^-$.

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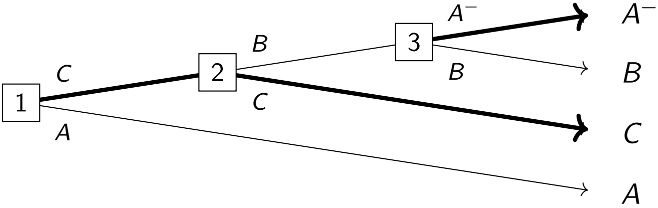

Node 1: accepting leads to C. Accept, because $C \succ A$.

Foresight in the Standard Pump

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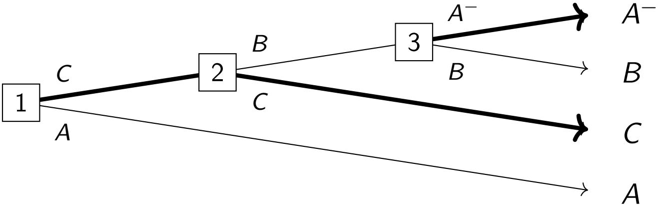
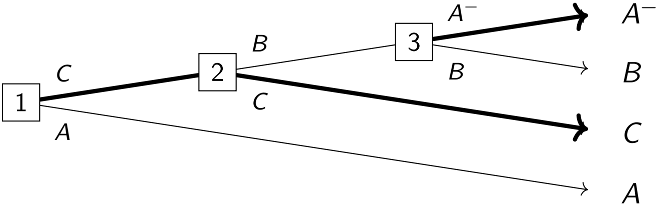


- Accept the first trade, from A to C.

Foresight in the Standard Pump

$$A \succ A^- \succ B \succ C \succ A, \text{ and } C \succ A^-.$$



- Refuse the second trade. It would lead through B to A^-.

Foresight in the Standard Pump

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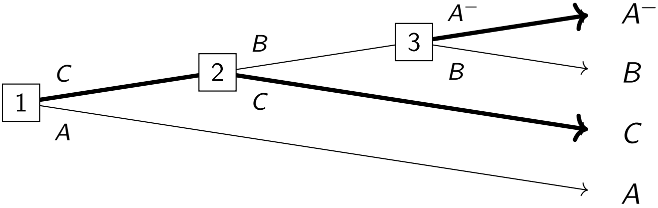

- Finish with C, which you prefer to your initial holding A. No payment.

Foresight in the Standard Pump

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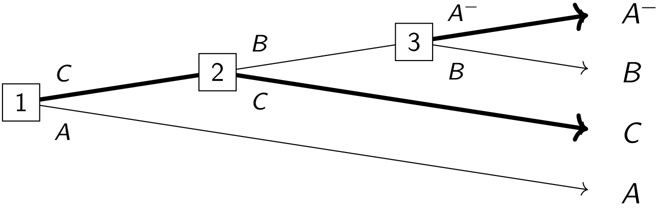
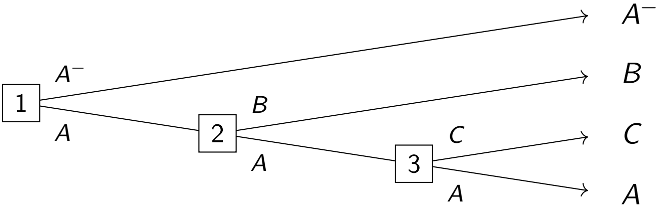


- The standard pump fails against this chooser.

The Gustafsson–Rabinowicz Pump

Start with A. Preferences: $A \succ A^- \succ B \succ C \succ A$.



- First, pay a small fee to keep A and end the offers: receive A^-.

Gustafsson (2022: 12), Figure 4.

The Gustafsson–Rabinowicz Pump

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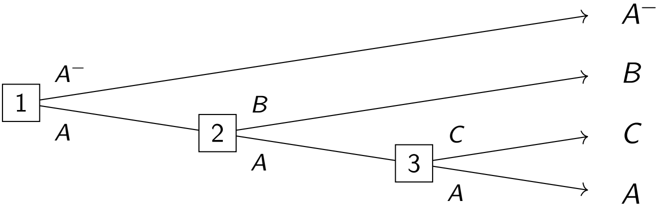

- If you refuse, you may trade A for B and stop.

Gustafsson (2022: 12), Figure 4.

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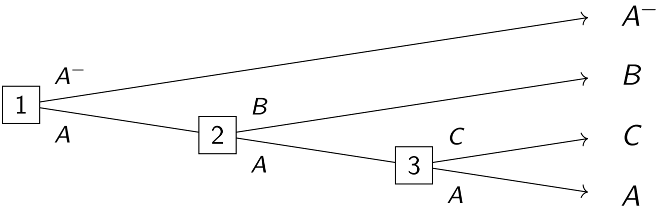

- If you refuse again, you may trade A for C and stop.

Gustafsson (2022: 12), Figure 4.

The Gustafsson–Rabinowicz Pump

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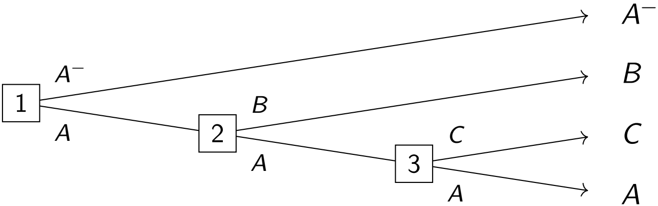

- Refuse all three offers and keep A for free.

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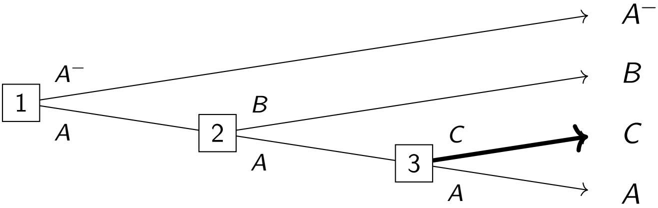

Work backwards. Node 3: accept C, because $C \succ A$.

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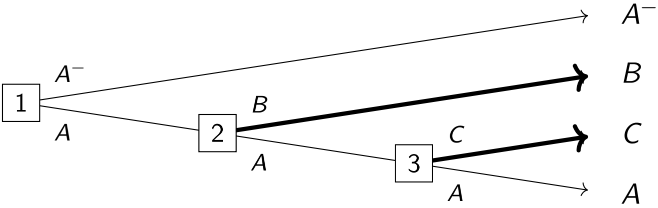

Node 2: refusing leads to C. Accept B, because $B \succ C$.

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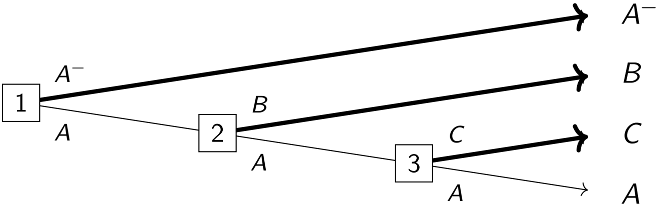

Node 1: refusing leads to B. Pay for A^-, because $A^- \succ B$.

Gustafsson (2022: 12), Figure 4.

Why This Is a Money-Pump

- You begin with A and finish with A^-. The exploiter collects the fee.

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- You could have kept A for free by refusing every offer.
- You know the entire set-up. Your preferences never change.
- Given sophisticated choice and the stated preferences, you pay immediately.

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- The argument targets cyclic preferences, given the assumptions about small costs and dynamic choice.
- At node 1, paying can be the rational choice given those preferences. That is why the preference structure is the target.
- Ruling out a strict cycle is weaker than proving the full Transitivity axiom.
- The remaining question: must rational agents choose in this sophisticated way?

The Same Trick Against Independence

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- The upfront pump caught a sophisticated chooser with cyclic preferences.
- The same construction works against a violation of Independence.
- One kind of violation: $A \succ B$, and yet $BpC \succ ApC$.
- Adding the same chance of C to both reverses the ranking.

The Small Cost Again

$$A \succ B, \qquad BpC \succ ApC.$$

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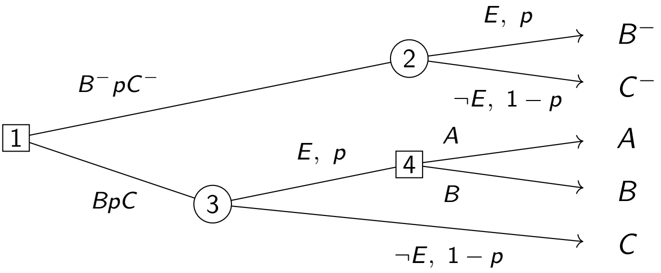
The Independence Money Pump

Start with BpC. $A \succ B$, and $BpC \succ B^{-}pC^{-} \succ ApC$.



Both chance nodes turn on the same event E, which has probability p.

Gustafsson (2022: 57), Figure 14.

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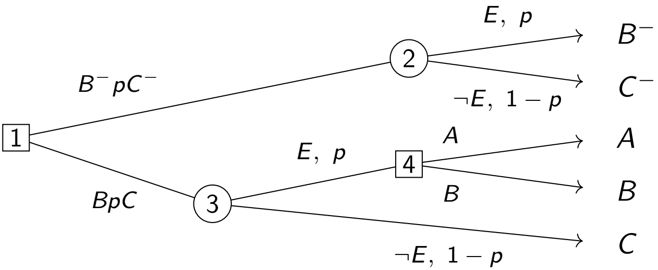

Start with the last choice, at node 4.

Gustafsson (2022: 57), Figure 14.

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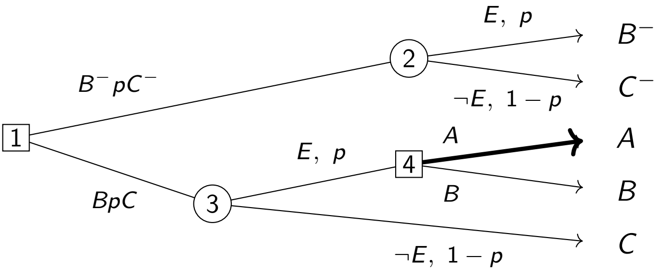

Node 4: trade B for A, because $A \succ B$.

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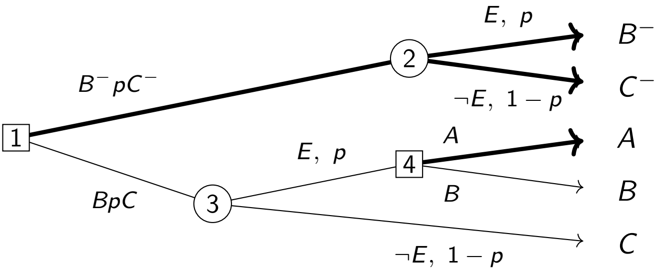

Node 1: going down is now effectively ApC. Going up is $B^{-}pC^{-}$. Go up.

Gustafsson (2022: 57), Figure 14.

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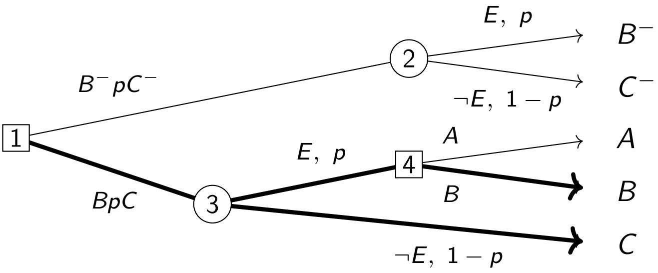

But going down at both choice nodes would have kept BpC, for free.

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This argument rules out strict preference reversals under mixing; establishing the full Independence axiom requires further argument.

A Dutch Book for Conditionalisation

Conditionalisation

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- Unpacking the definition of conditional probability: $\mathrm{Cr}(A \cap E) = \mathrm{Cr}(A \mid E)\ \mathrm{Cr}(E) = 0.3$.
- But your rule is: on learning E, move to $\mathrm{Cr}_{\mathrm{new}}(A) = 0.7$.

Lewis (1999), via Pettigrew (2020: 36–40).

Conditional Bets

A \$100 bet on A, **called off if** $\neg E$: pays \$100 if $A \cap E$; nothing if $\neg A \cap E$; if $\neg E$, the bet is cancelled and the price refunded.

	$A \cap E$	$\neg A \cap E$	$\neg E$
Sell a \$100 bet on A for \$60, called off if $\neg E$	-40	$+60$	0

Her price is $\$100 \times \mathrm{Cr}(A \mid E) = \$60$. Where does that come from?

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Sell a \$100 bet on A for \$60, called off if $\neg E$	-40	$+60$	0
Sell a \$100 bet on $A \cap E$ for \$30	-70	$+30$	$+30$

$\mathrm{Cr}(A \cap E) = 0.3$, so \$30 is her price for a \$100 bet on $A \cap E$.

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Buy a \$60 bet on E for \$30	$+30$	$+30$	-30

$\mathrm{Cr}(E) = 1/2$, so \$30 is her price for a \$60 bet on E.

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Sell a \$100 bet on $A \cap E$ for \$30	-70	$+30$	$+30$
Buy a \$60 bet on E for \$30	$+30$	$+30$	-30
Together	-40	$+60$	0

Two ordinary bets, each fair by her lights. Together they are exactly the conditional bet at \$60.

Justifying the Conditional Bet

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Justifying the Conditional Bet

- **Route 1: bundling.** If you would take each of two bets on its own, you should take both together. Then she takes the conditional bet, at any price above $60.
- That is an extra assumption, and not an innocent one: it says that bets never interact.
- **Route 2: Independence.** No bundling principle at all. The comparison we want has exactly the shape Independence speaks to.

Independence and Conditional Bets

	E, probability $1/2$	$\neg E$, probability $1/2$
Take the conditional bet	X	Z
Don't	Z	Z

The two prospects

Z: no bet.

X: sell a \$100 bet on A for \$61, assessed *given* E.

Net payoff	$-\$39$	$+\$61$
Probability	0.6	0.4

Independence and Conditional Bets

	E, probability $1/2$	$\neg E$, probability $1/2$
Take the conditional bet	X	Z
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Why prefer X to Z?

The conditional credences $\mathrm{Cr}(\cdot \mid E)$ form a credence function (week 4).

With credence 0.6 in A, the betting interpretation says to sell the \$100 bet for \$61.

$$X \succ Z.$$

Independence and Conditional Bets

	E, probability $1/2$	$\neg E$, probability $1/2$
Take the conditional bet	X	Z
Don't	Z	Z

Apply Independence

Both options in the table give Z if $\neg E$.

$$X \succ Z \quad \Longleftrightarrow \quad X \frac{1}{2} Z \succ Z \frac{1}{2} Z.$$

So she prefers taking the conditional bet to not taking it.

Independence and Conditional Bets

	E, probability $1/2$	$\neg E$, probability $1/2$
Take the conditional bet	X	Z
Don't	Z	Z

Recover the conditional bet's payoffs

Reduce the compound lottery $X \, \frac{1}{2} \, Z$:

Net payoff	$-\$39$	$+\$61$	$\$0$
Probability	$\frac{1}{2}(0.6) = 0.3$	$\frac{1}{2}(0.4) = 0.2$	0.5

These are exactly the payoffs and probabilities of Bet 1.

Three Bets

Bet 1 Now: you **sell** a \$100 bet on A, called off if $\neg E$, for \$61. Your price is \$60, so selling above it is a gain.

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Each one, when it is offered, is a bet you want.

Combining the Three Bets

	$A \cap E$	$\neg A \cap E$	$\neg E$
Bet 1	-39	$+61$	0

Net gains in dollars. Bet 1: sold for \$61; pays out \$100 if A; called off if $\neg E$.

Combining the Three Bets

	$A \cap E$	$\neg A \cap E$	$\neg E$
Bet 1	-39	$+61$	0
Bet 2	$+5$	$+5$	-3

Bet 2: sold for \$5; pays out \$8 if $\neg E$.

Combining the Three Bets

	$A \cap E$	$\neg A \cap E$	$\neg E$
Bet 1	-39	$+61$	0
Bet 2	$+5$	$+5$	-3
Bet 3	$+31$	-69	not offered

Bet 3, only if E: bought for \$69; pays \$100 if A.

Combining the Three Bets

	$A \cap E$	$\neg A \cap E$	$\neg E$
Bet 1	-39	$+61$	0
Bet 2	$+5$	$+5$	-3
Bet 3	$+31$	-69	not offered
Total	-3	-3	-3

Down \$3, however things turn out.

A Strategy, Not a Book

- Bet 3 is offered only if E occurs, and at your *new* price.

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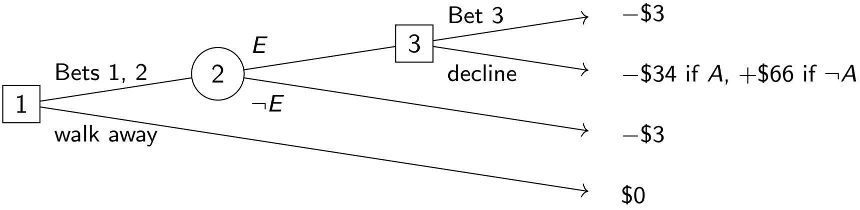
- So the bookie needs to know your updating rule in advance. The target is the rule.
- If instead your rule moved you below 0.6, reverse every bet.
- Any rule other than conditionalisation can be booked this way: a *Dutch strategy*.

Applying Sophisticated Choice

Net gains in dollars. Take the earlier bets, or walk away.



Work backwards.

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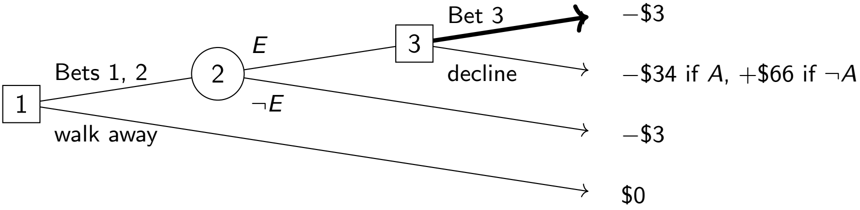
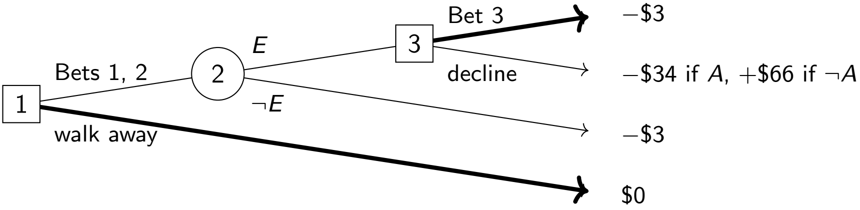


Node 3: your credence is then 0.7, so you take Bet 3. Down $3 either way.

Applying Sophisticated Choice

Net gains in dollars. Take the earlier bets, or walk away.



Node 1: taking the bets leads to −$3 in every state. Walk away.

Skyrms's Reply: The Later Offer Remains

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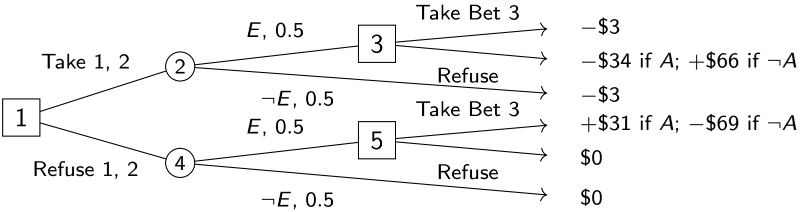

The previous tree let you end all offers by refusing now.

Skyrms's reply: **Bet 3 is offered after *E* even if you refuse Bets 1 and 2.**

The lower branch now leads to another choice. Terminal amounts are *total* net payoffs.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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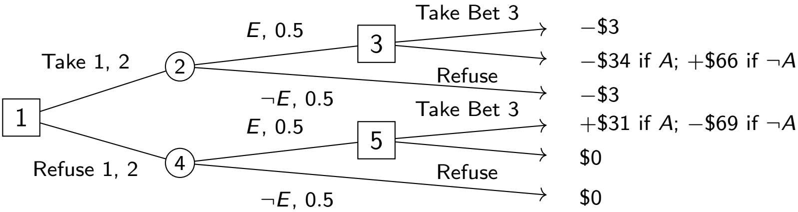

Work backwards from the choices after E.

At both node 3 and node 5, your credence in A is then 0.7.

Bet 3 costs \$69 and pays \$100 if A: its net payoffs are $+\$31$ if A and $-\$69$ if $\neg A$.

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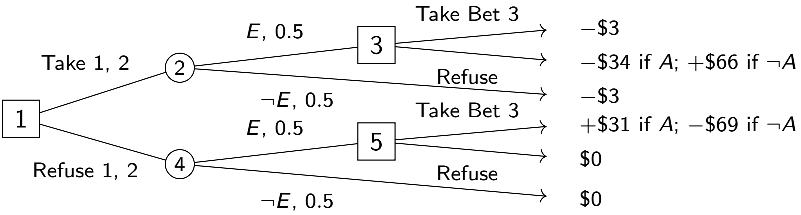

Node 3: you already hold Bets 1 and 2.

Refusing Bet 3 leaves $-\$34$ if A and $+\$66$ if $\neg A$:

$$0.7(-34) + 0.3(66) = -4.$$

Taking Bet 3 instead leaves $-\$3$ whichever occurs.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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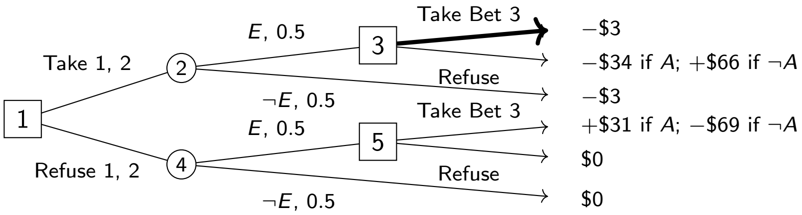

Node 3: take Bet 3.

You prefer a sure −\$3 to an expected −\$4.

The upper final choice is now bold. This is what you predict you would do if you reached node 3.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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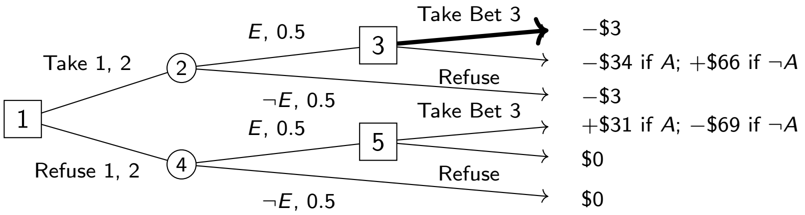

Node 5: you refused Bets 1 and 2, so you hold no bets.

Refuse Bet 3: \$0. Take it:

$$0.7(31) + 0.3(-69) = 21.7 - 20.7 = +1.$$

By your *new* credences, Bet 3 has a positive expected payoff.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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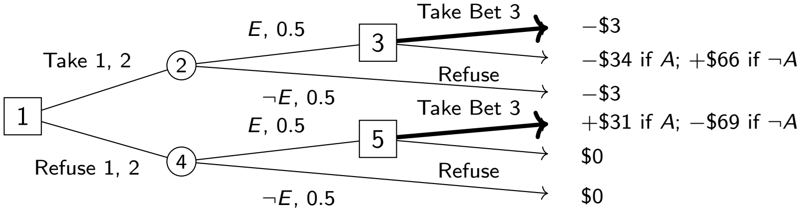

Node 5: take Bet 3 as well.

You prefer an expected +\$1 to \$0.

Both later choices are now bold. Refusing the first offers does *not* mean refusing the later offer.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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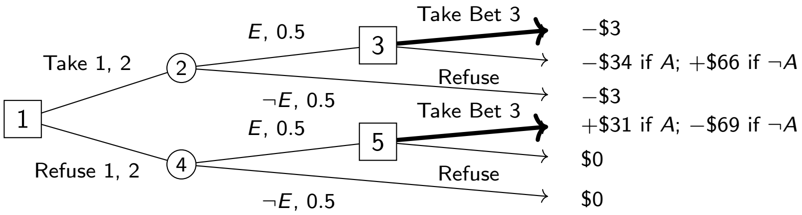

Return to node 1: evaluate those predicted choices now.

Your current conditional credence is $\mathrm{Cr}(A \mid E) = 0.6$, not 0.7.

Use 0.7 to predict the later choice; use your *current* credences to evaluate its consequences.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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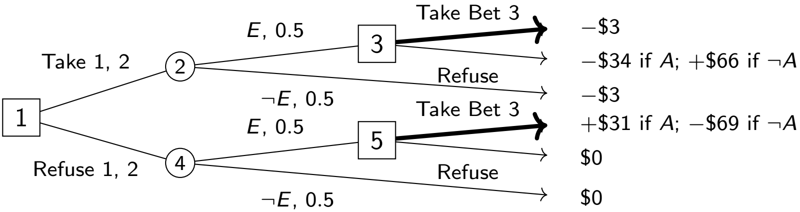

The three cases, by your current credences:

Case	$A \cap E$	$\neg A \cap E$	$\neg E$
Credence	0.3	0.2	0.5

For example, $\mathrm{Cr}(A \cap E) = \mathrm{Cr}(E)\,\mathrm{Cr}(A \mid E) = \frac{1}{2}(0.6) = 0.3$.

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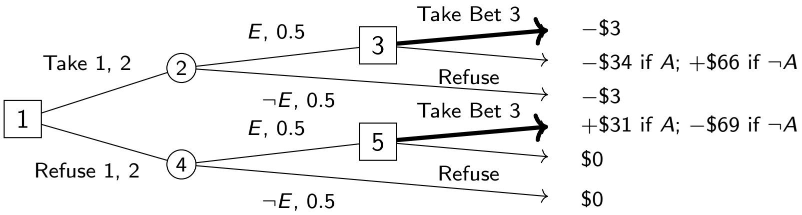

Suppose you go down at node 1.

You refuse Bets 1 and 2. If $\neg E$ occurs (chance 0.5), no further bet is offered.

A 0.5 **chance of** $0.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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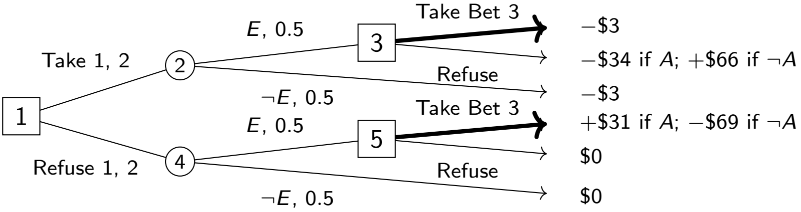

Down at node 1; then E and $\neg A$.

At node 5 you take Bet 3, paying \$69. It pays nothing because A is false.

A 0.2 **chance of** $-\$69$**.**

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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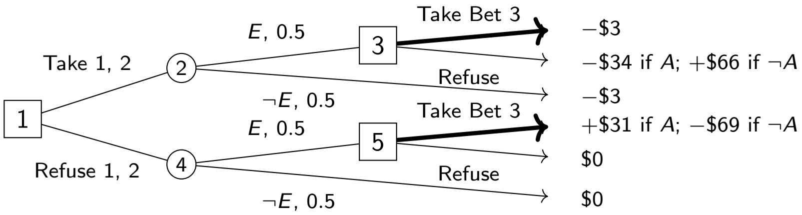

Down at node 1; then E and A.

At node 5 you take Bet 3, paying \$69. It pays \$100 because A is true.

A 0.3 **chance of** $+\$31$**.**

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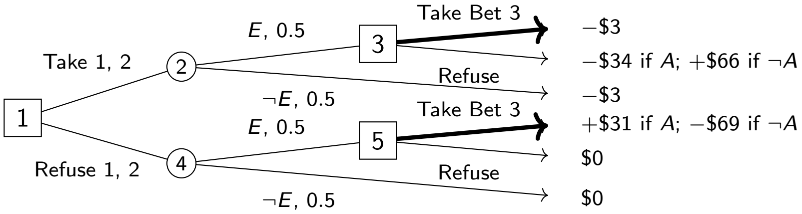

The prospect from going down:

Payoff	$0	−$69	+$31
Probability	0.5	0.2	0.3

These are the consequences of refusing now *and taking Bet 3 later if E*.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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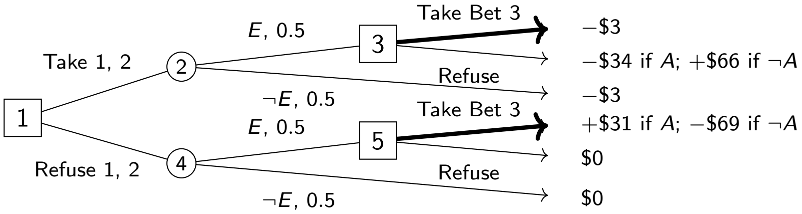

Expected payoff from going down, assessed now:

$$0.5(0) + 0.2(-69) + 0.3(31) = -13.8 + 9.3 = \mathbf{-4.5}.$$

Your present self regards the bet your future self will take as a bad deal.

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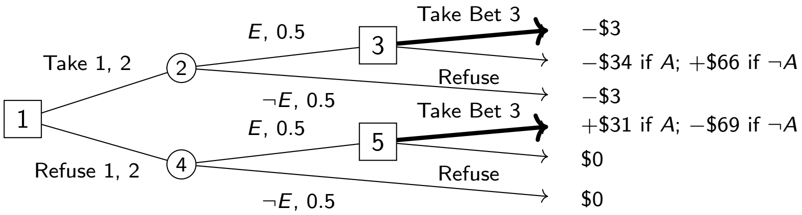

Suppose you go up at node 1.

Take Bets 1 and 2. If E occurs, you then take Bet 3 at node 3.

If $\neg E$ occurs, Bet 1 is cancelled and Bet 2 loses \$3.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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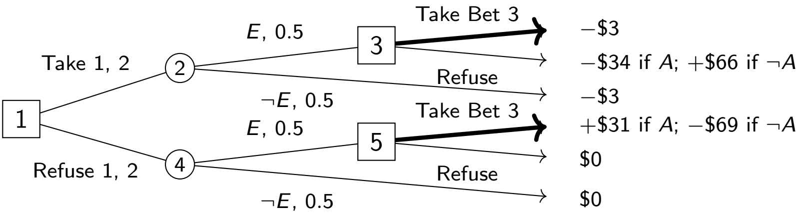

Up at node 1: combine the bets in each case.

	$A \cap E$	$\neg A \cap E$	$\neg E$
Total	$-39 + 5 + 31$	$61 + 5 - 69$	$0 - 3$
	-3	-3	-3

A **certainty of** $-\$3$.

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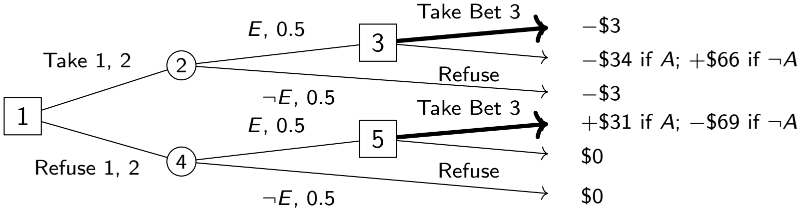

Compare your two options at node 1.

Take Bets 1 and 2	$-\$3$ for sure
Refuse Bets 1 and 2	$-\$4.50$ in expectation

Both are negative. Going up is better by your current credences.

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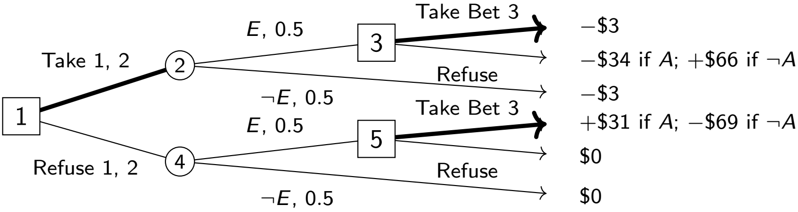

Node 1: take Bets 1 and 2.

The initial choice is now bold too. You foresee taking Bet 3 if E, and choose the sure loss of \$3.

Foresight does not let you escape this tree.

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

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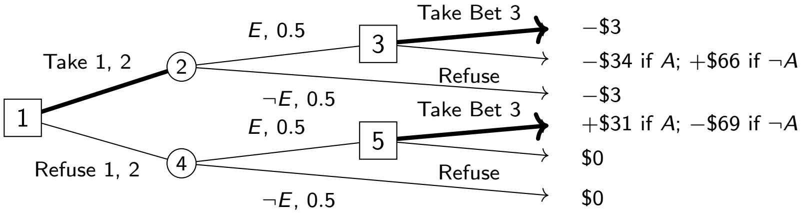

Why the earlier escape failed.

Refusing *now* no longer secures \$0. It leaves you exposed to a later offer that you predict you will accept.

The bookie need not go away when you decline the first transaction.

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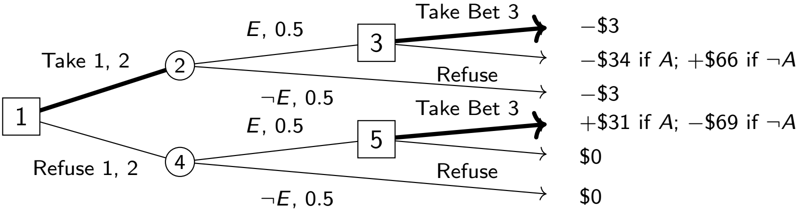

What about refusing every bet?

That plan would preserve \$0. But following it requires refusing Bet 3 at node 5, where your new credences favour taking it.

This brings us to *resolute choice*: can you rationally stick to the earlier plan?

Adapted from Skyrms (1993: 323–326), Figure 2; using our three bets.

Odysseus and the Sirens

(or: Fun with Ropes)

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- He has anticipated a later demand and arranged for it not to be obeyed.

Commitment and Resolve

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- Now imagine following the plan without ropes: you remain able to deviate, but carry it through.
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- The question is whether that can be rational when you prefer to deviate.

Planning

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Carry out an adopted plan even when, without that plan, you would choose differently at a later node.

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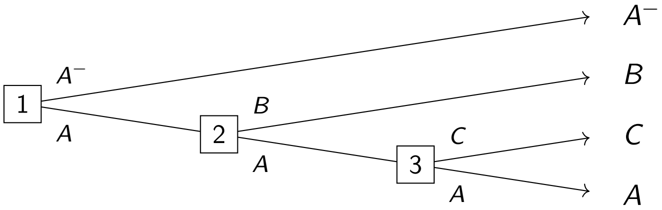
- The resolute chooser follows through.

McClennen (1985; 1990).

Resolute Choice in the Upfront Pump

Adopt the plan: refuse all three offers.



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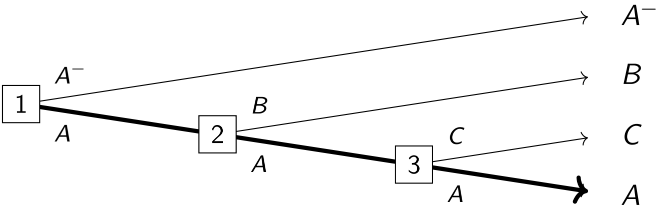

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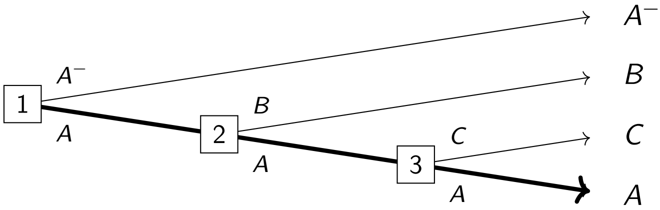

Follow the thick route to A. At node 3, keep A despite $C \succ A$.

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- Resolute choice rejects that prediction and, on the counter-preferential version, the local choice rule behind it.

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The General Defence

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- Adopt that plan and reliably carry it out, including at later nodes where you would otherwise deviate.
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The same strategy can block any money pump with such an available plan.

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- Being able to avoid exploitation is one thing. Doing so rationally is another.
- Does an earlier plan give you a reason to reject an option you now prefer?
- Or should rationality assess the whole plan, rather than each choice in isolation?

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- The resolute response: the choice implements a plan whose adoption avoids the loss.
- The dispute concerns whether that earlier rationale still governs the later choice.

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- Then you can refuse the offers without choosing against your current preferences.
- But if escaping requires abandoning the original cyclic preferences, have those preferences been defended?
- Avoiding the pump need not refute the argument against the original preferences.

Removing the Later Options

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- That changes the available options. The original pump allowed you to accept each offer.
- A commitment device may be unavailable or costly.
- Resoluteness as a choice policy has to be distinguished from physically changing the decision problem.

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- Then the outcomes need more detailed descriptions. The original preference comparisons may no longer apply.
- Gustafsson asks why keeping this plan should matter in itself.
- That is a substantive dispute about what it is rational to care about.

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- Once that opportunity is gone, does it still give a reason to follow the plan?
- Or does treating the later choice in isolation miss what makes planned action rational?
- Decision-tree separability is now part of the dispute, rather than common ground.

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Where the Argument Stands

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What should rationality govern: preferences, individual choices, or whole plans?

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- I will try to provide more materials soon. The existing lecture notes may also be useful for preparation.