Decision and Social Choice

PE3101P — Lecture 9: Calibration

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Today

1. Introduction
2. Calibration and Risk
3. Calibration and Distribution
4. Gini Egalitarianism
5. Calibration Without Numbers
6. Ways Out

An Extra Dollar

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- Prioritarians care about the first. Egalitarians care about the second. Virtually everybody cares about the third.

Diminishing Marginal Value

- If I have no money at all, a dollar might buy me a bag of rice, which would matter a lot if I have no other food.

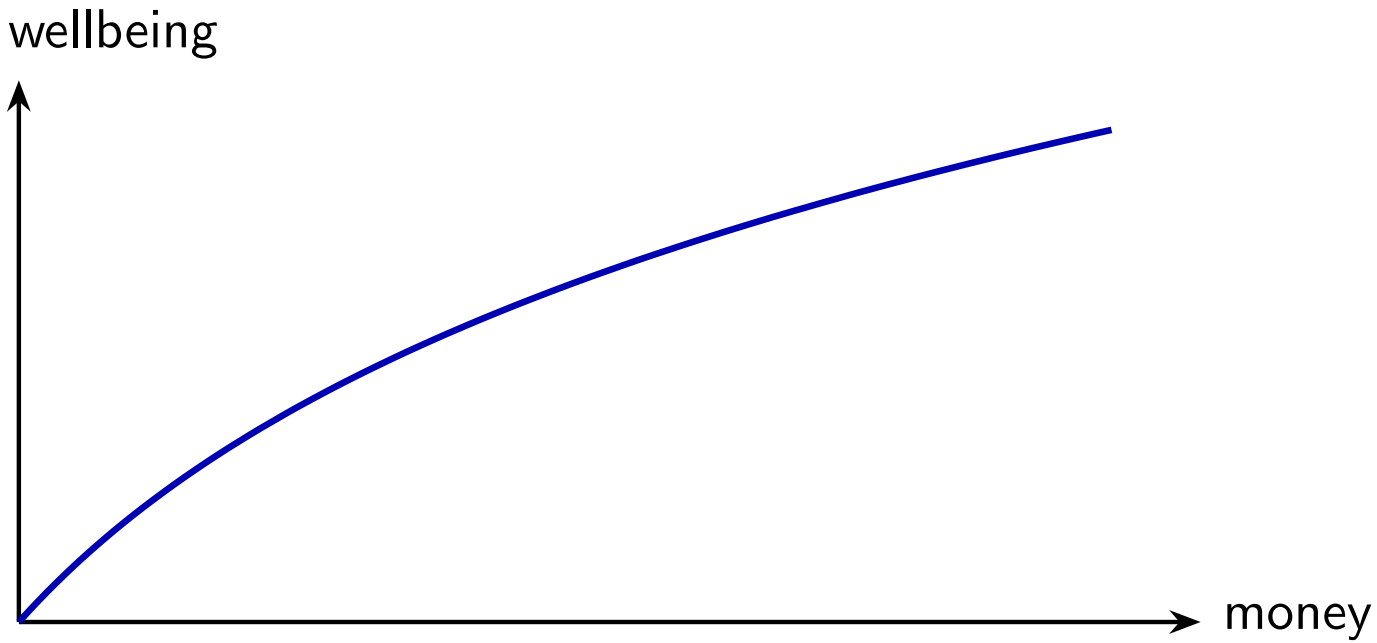
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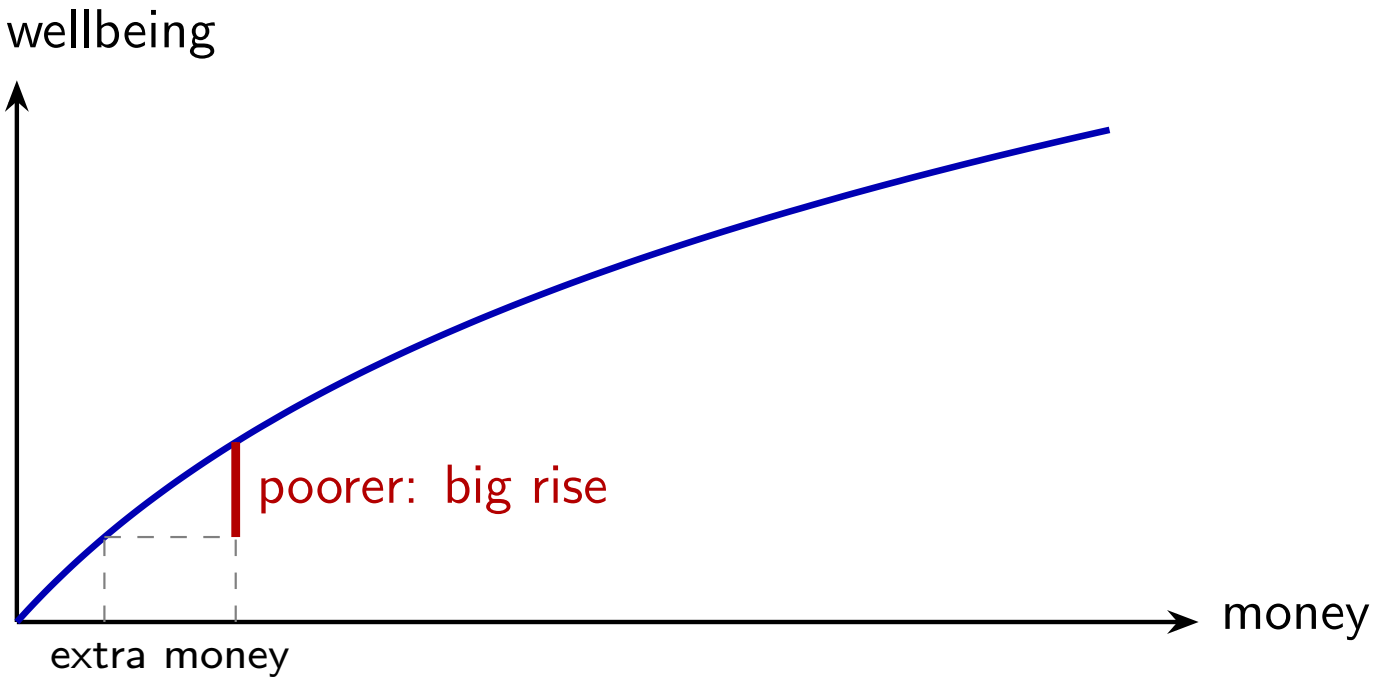
- If I have no money at all, a dollar might buy me a bag of rice, which would matter a lot if I have no other food.
- If I have a million dollars, I can already buy all the rice I can eat.
- Many utilitarians think this explains our equality-favouring judgements about resources, without our needing to say that equality matters in itself.

The Same Money at Two Wealth Levels



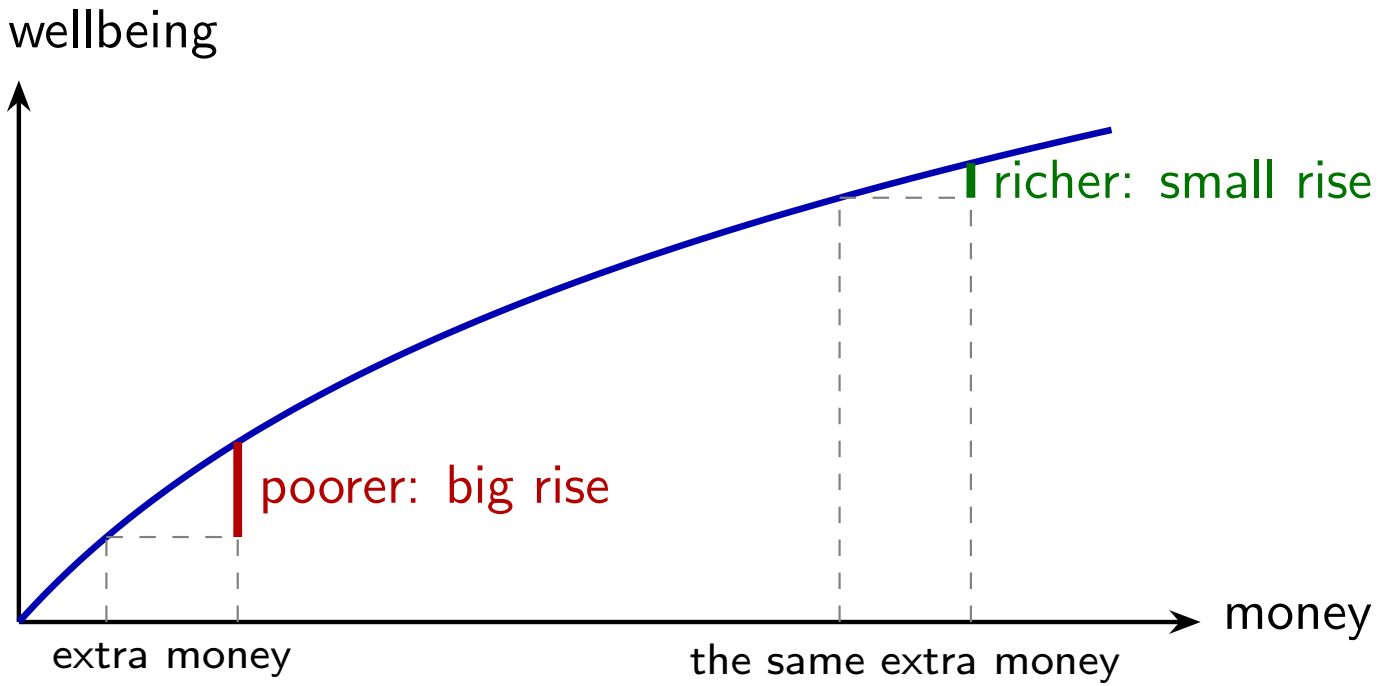
Wellbeing against money: each extra dollar adds less wellbeing than the one before.

The Same Money at Two Wealth Levels



Some extra money for someone poorer: a big rise in wellbeing.

The Same Money at Two Wealth Levels



The same money for someone richer: a small rise.
So it does more for total wellbeing if the poorer person gets it.

The Calibration Problem

- Roughly: if we favour equality moderately in low-stakes cases, where not much money or wellbeing is at stake, then we have to favour equality to an implausibly extreme extent in high-stakes cases.

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- The calibration problem undermines the utilitarian explanation, and also many views on which equality or priority to the worse off *is* significant in itself.
- The problem arises for risk as well as for distribution, because states of nature play the role that people play.

Where the Calibration Problem Bites

1. **Risk.** Risk aversion, in preferences under risk.

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- So it's an issue for everybody, not just for egalitarians and prioritarians.

Would You Take This Gamble? (1)

A 50–50 gamble:

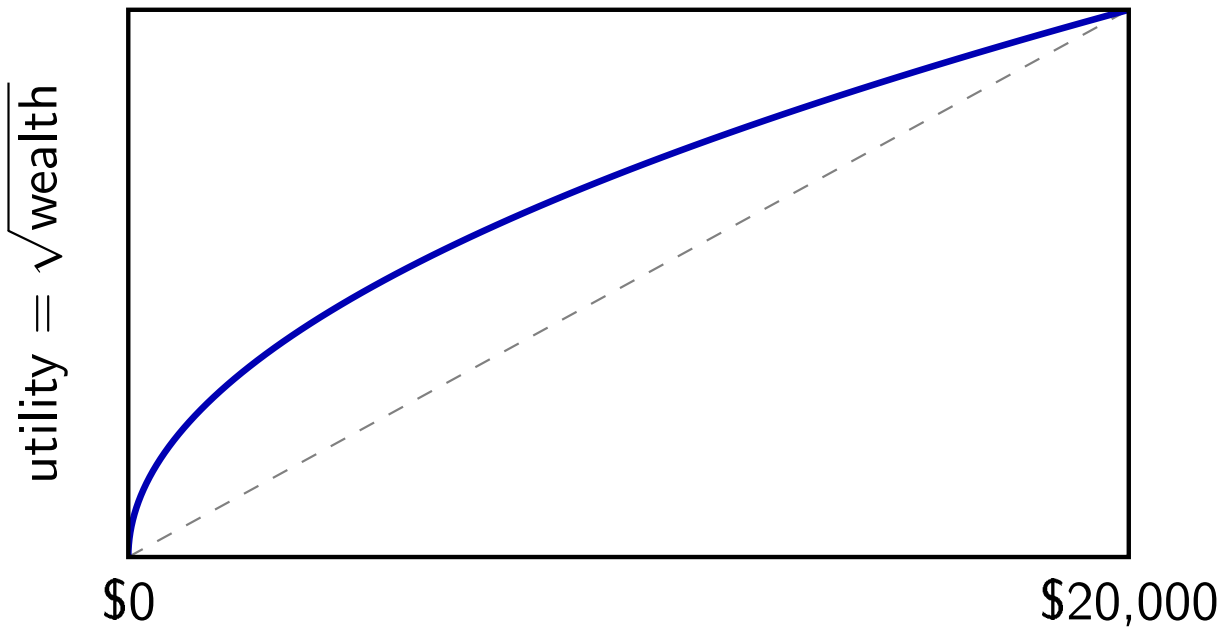
gain \$1,100 or lose \$1,000.

Would You Take This Gamble? (2)

A 50–50 gamble:

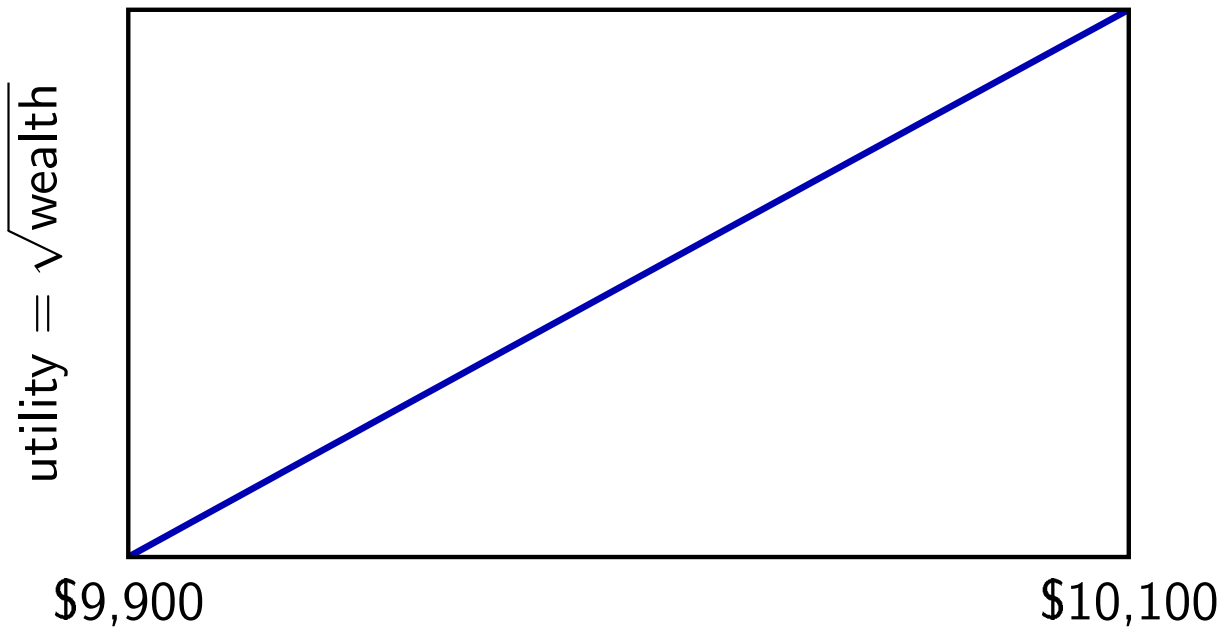
gain $1,000,000,000 or lose $7,000.

Zooming In on a Concave Curve



Utility = $\sqrt{\text{wealth}}$, from \$0 to \$20,000.
Clearly curved.

Zooming In on a Concave Curve



From $9,900 to $10,100.
Almost a straight line.

Less Risk Averse, the Closer You Look

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- Zoom in on a concave curve and it gets flatter and flatter, closer and closer to a straight line. So for small gambles, it is almost risk neutral.
- So if someone is still risk averse when we zoom in a lot, their curve must still bend at that small scale.
- Then, zoomed out, it bends a lot: they are *extremely* risk averse about large gambles.

Square Root at $10,000

- Someone whose utility is $\sqrt{\text{wealth}}$ has $10,000: utility 100.
- A 50–50 gamble is worth taking if the utility they'd gain if they win is more than the utility they'd lose if they lose.

50–50 gamble	Utility lost	Utility gained

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50–50 gamble	Utility lost	Utility gained
Lose \$1,900 / gain \$2,000	$100 - 90 = 10$	less than $110 - 100 = 10$

Losing \$1,900 leaves \$8,100: utility 90.
Gaining \$2,000 gives \$12,000, less than \$12,100: utility less than 110.
So the loss outweighs the gain: they turn it down.

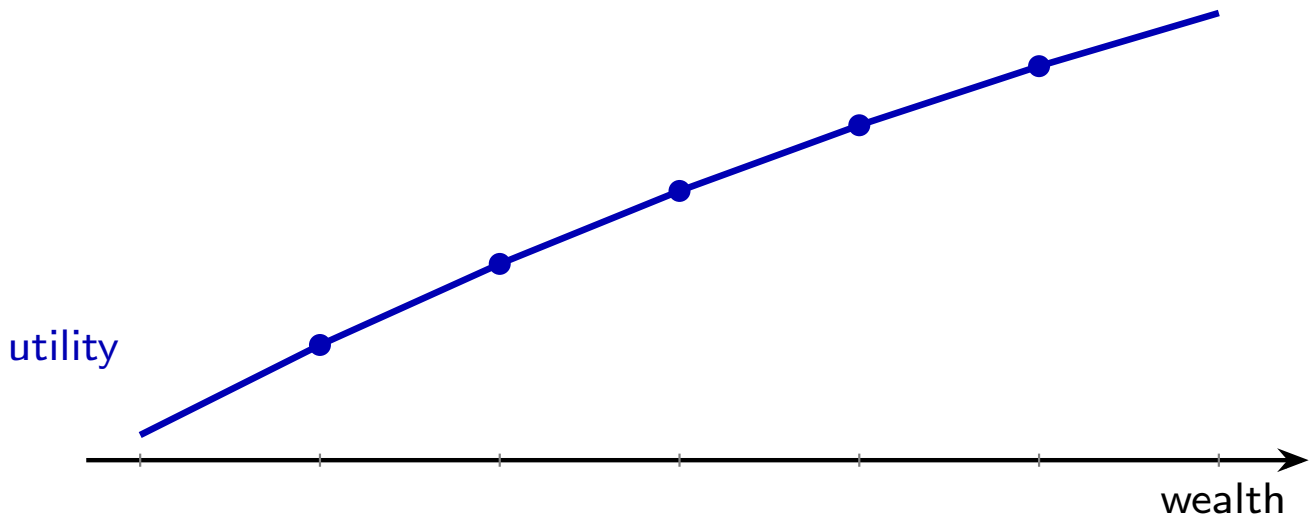
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Lose \$1,900 / gain \$2,000	$100 - 90 = 10$	less than $110 - 100 = 10$
Lose \$1,000 / gain \$1,100	about 5.1	about 5.4

Over \$1,000 the curve is almost straight:
gaining \$1,100 beats losing \$1,000, so they take it.
Square root isn't risk averse enough to turn down small gambles like this.

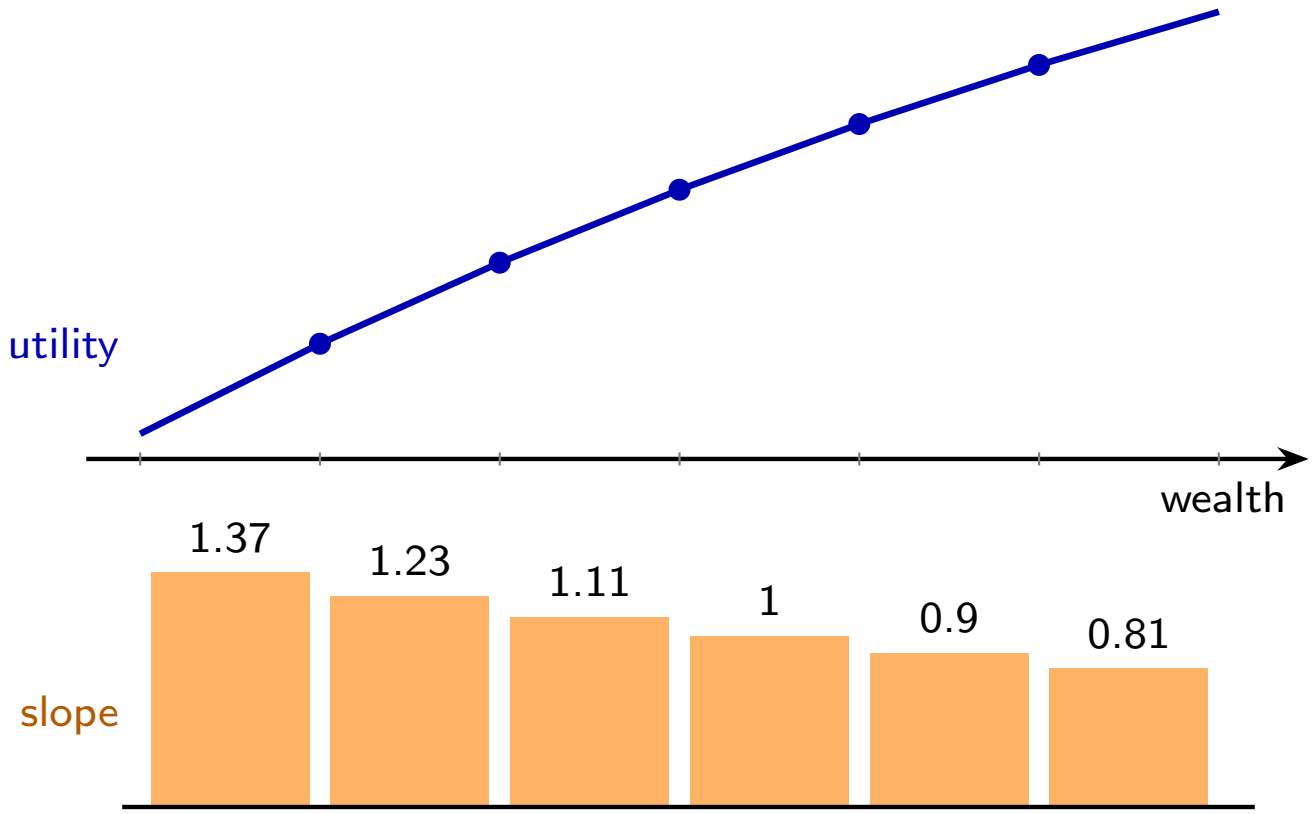
A Utility Function Made of Straight Pieces



Suppose someone turns down lose \$1,000 / gain \$1,100 at every wealth level.

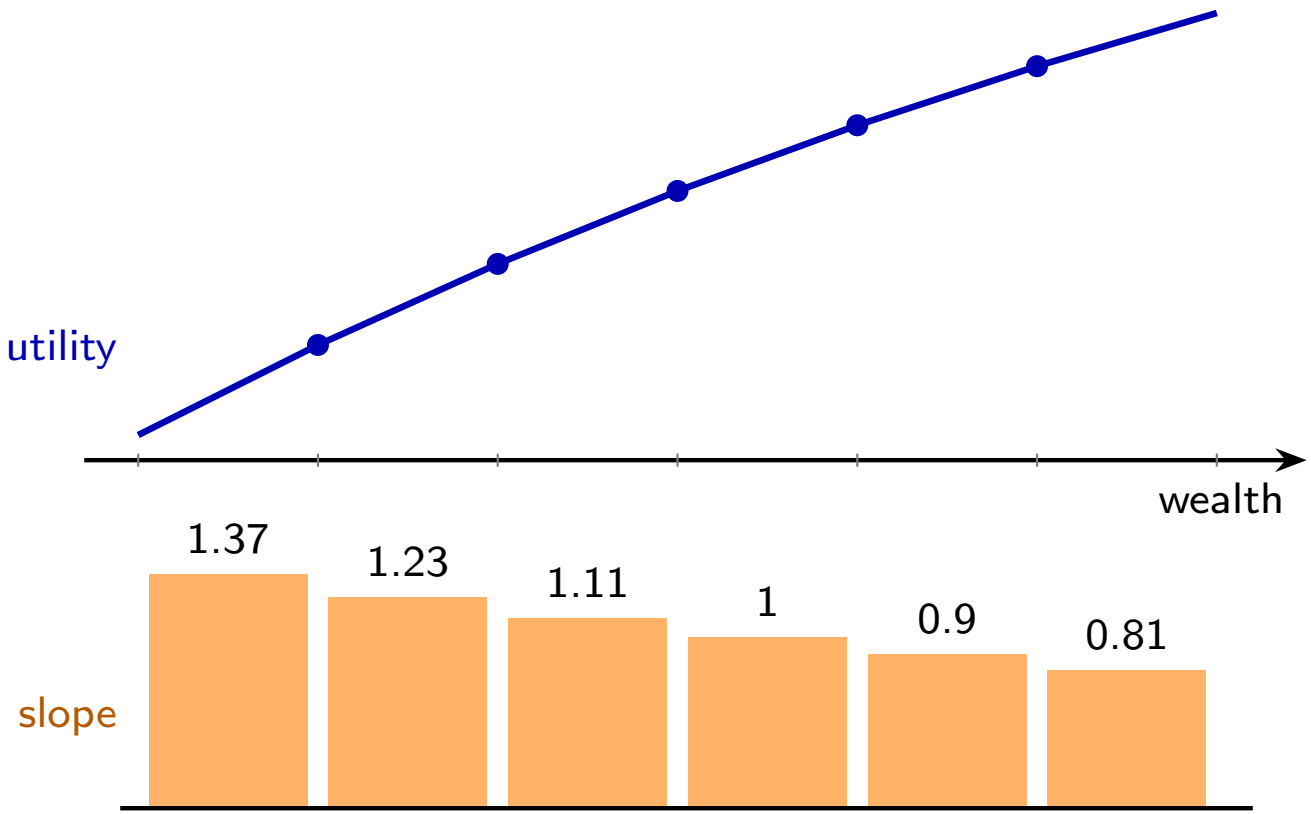
Here is one utility function that does that: straight \$1,000 pieces.

A Utility Function Made of Straight Pieces



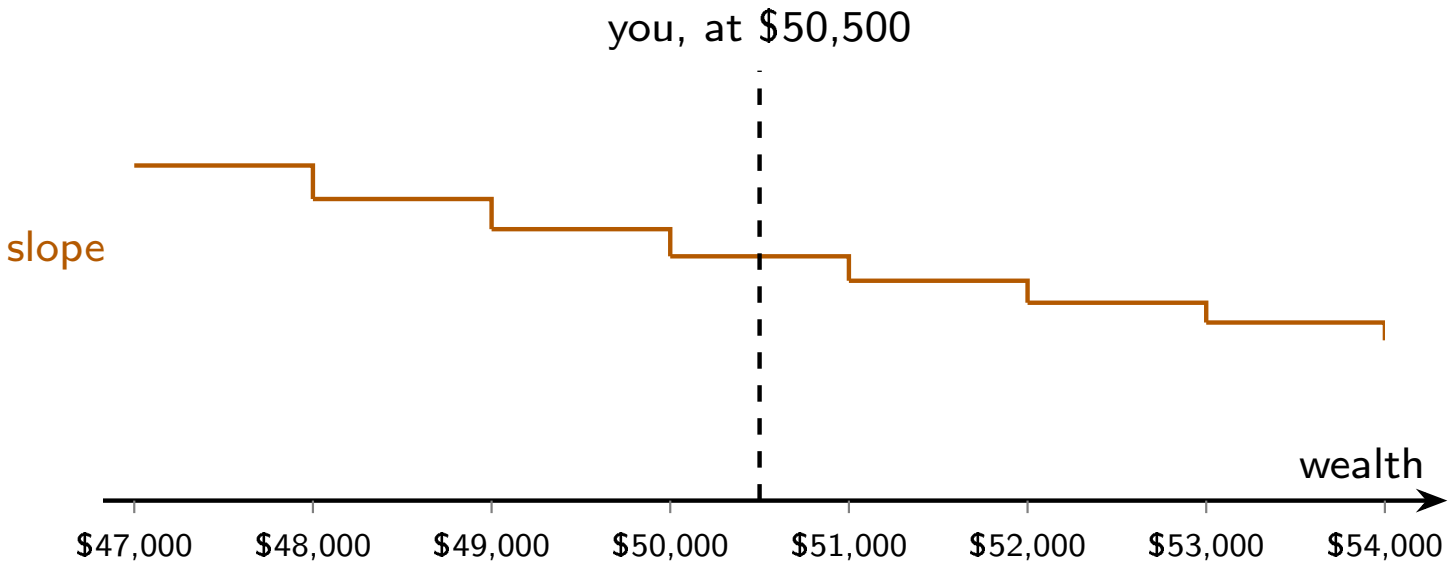
Below: the slope of each piece.
The slope is how much utility each extra dollar adds at that point.

A Utility Function Made of Straight Pieces



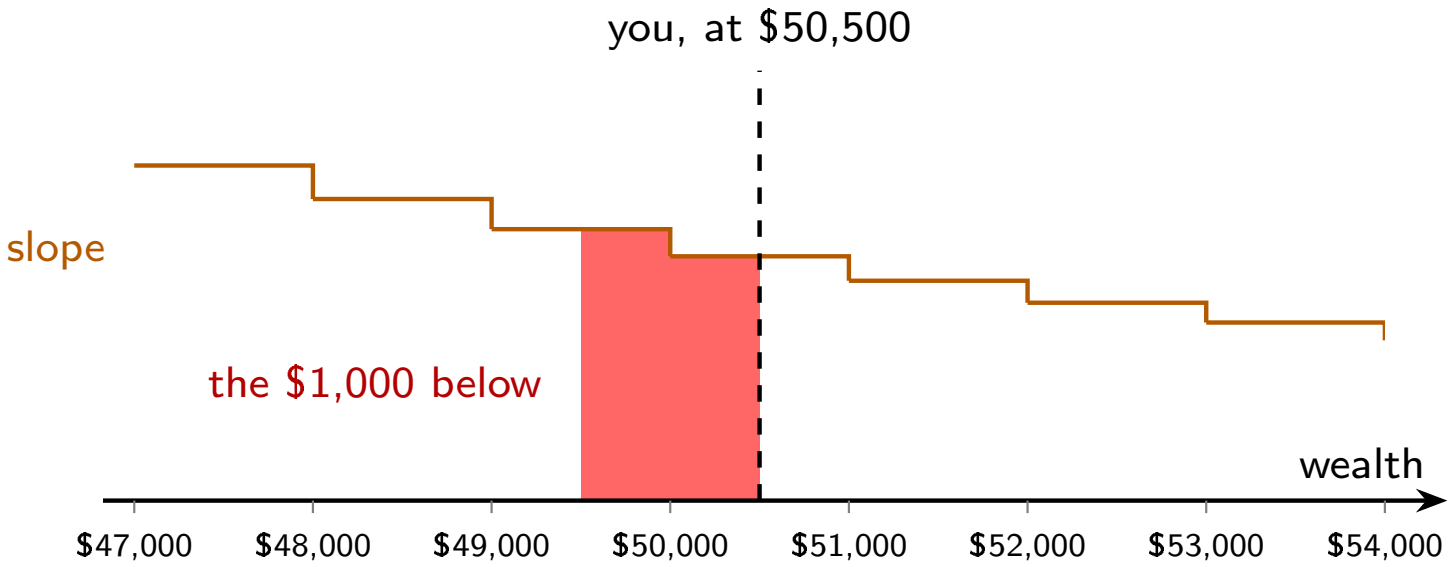
Every \$1,000, the slope is multiplied by 0.9.

Diminishing Marginal Utility



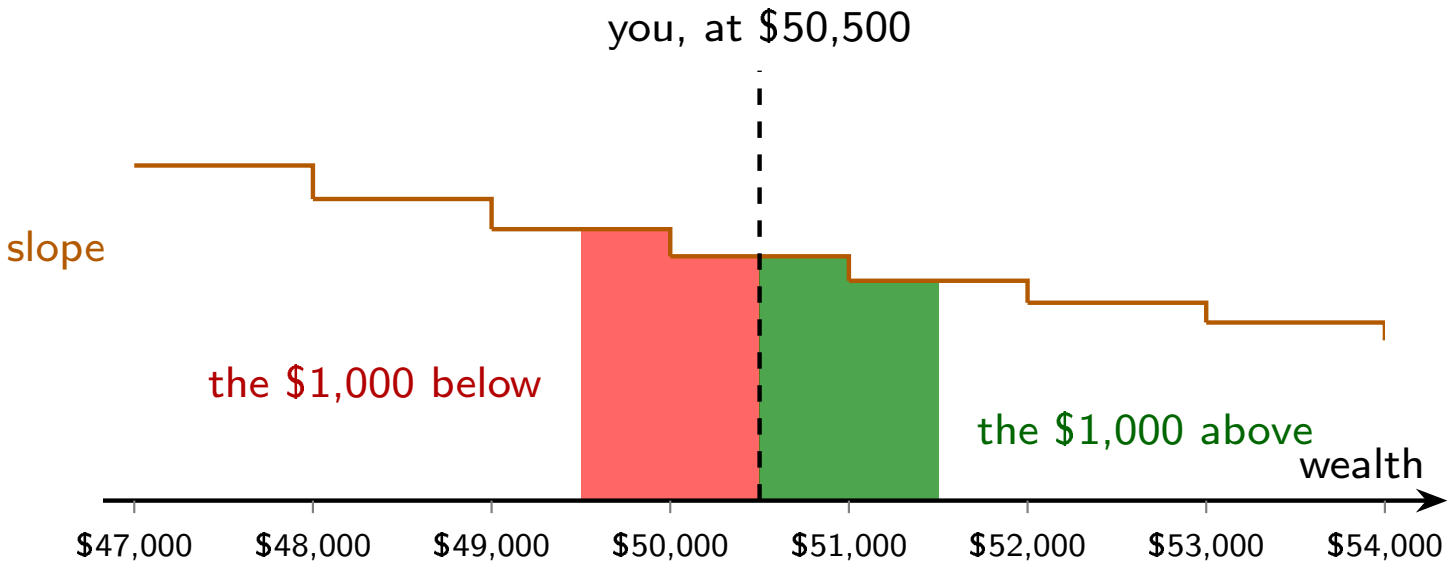
The slope again, with real wealth levels: you have $50,500, in the middle of a piece.

Diminishing Marginal Utility



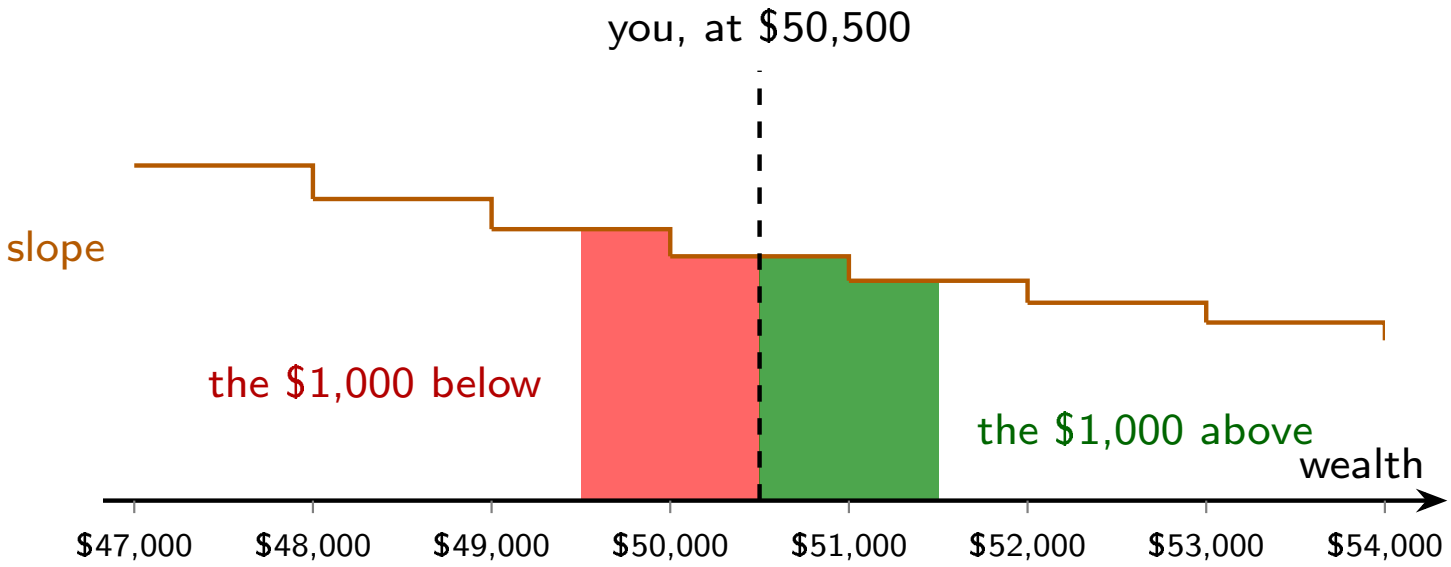
Red: what the $1,000 below you is worth, the area under the slope.

Diminishing Marginal Utility



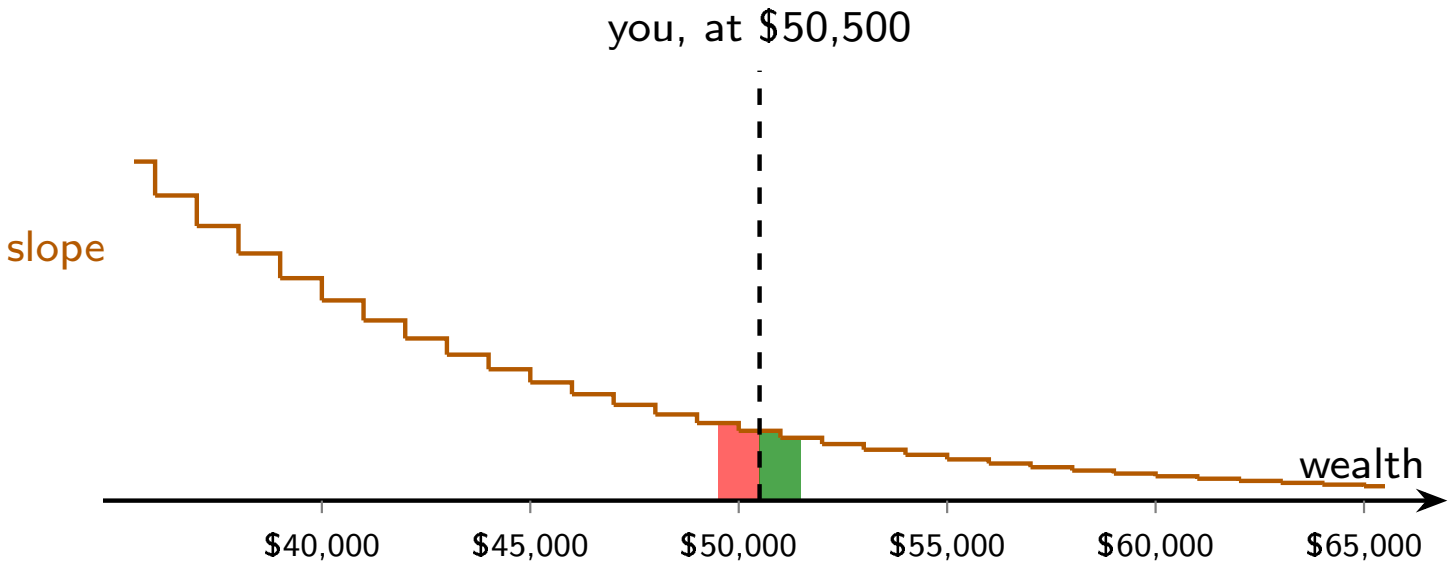
Green: what the $1,000 above you is worth.
It's the red area moved up $1,000, where every slope is 0.9 as big.
So it's worth 0.9 of the red.

Diminishing Marginal Utility



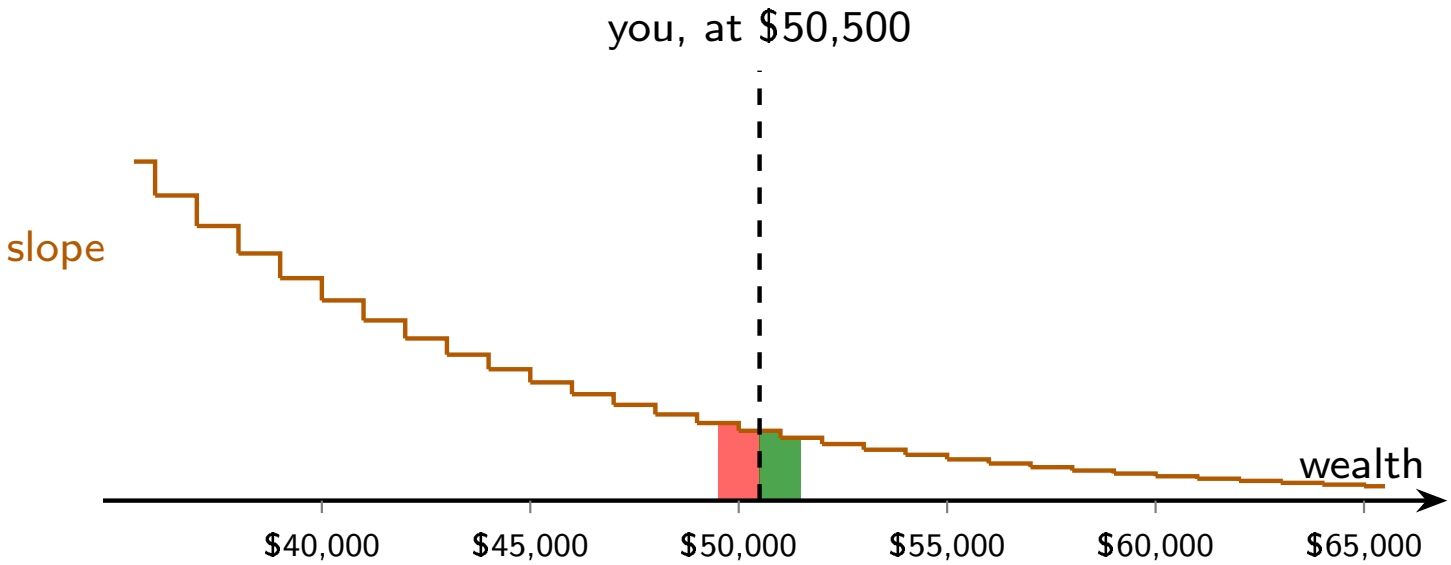
So gaining \$1,100 is worth at most $0.9 + 0.09 = 0.99$ of losing \$1,000.
That's why we chose 0.9: it's the kind of drop-off needed
to turn down 50–50 lose \$1,000 / gain \$1,100, wherever you start.

Diminishing Marginal Utility



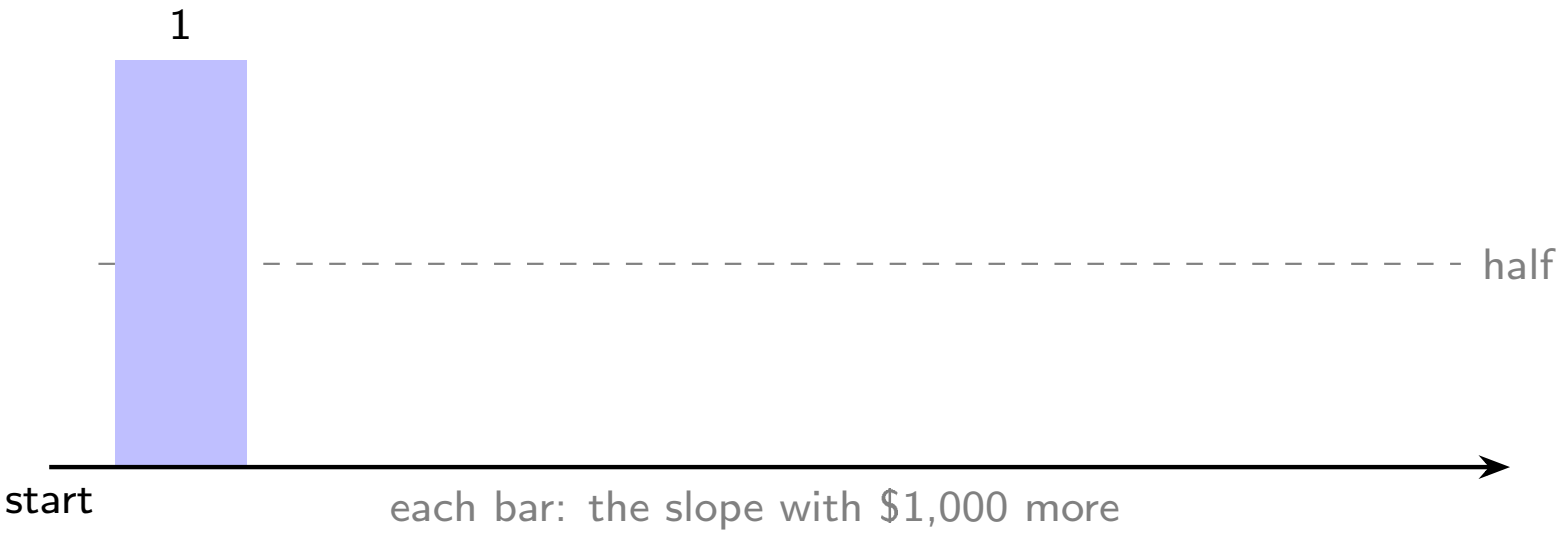
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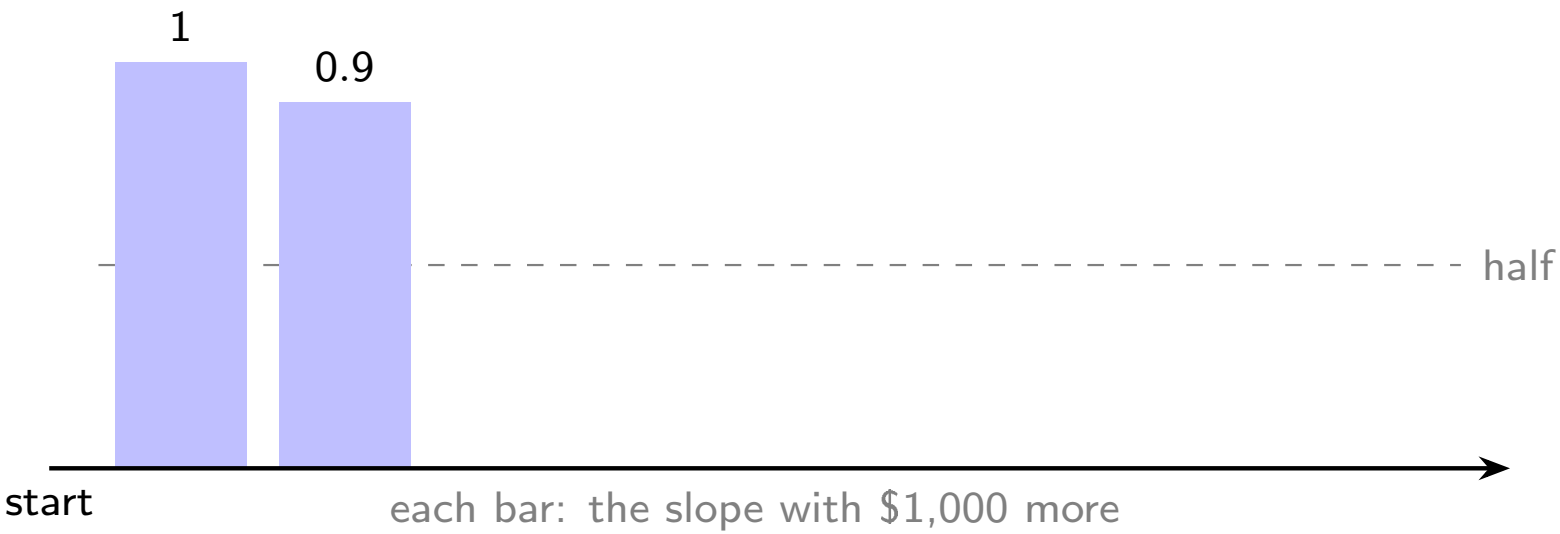
Zoomed out, from about $35,000 to $65,000:
the less wealth, the higher and higher the slope.

Seven Steps Halve the Slope



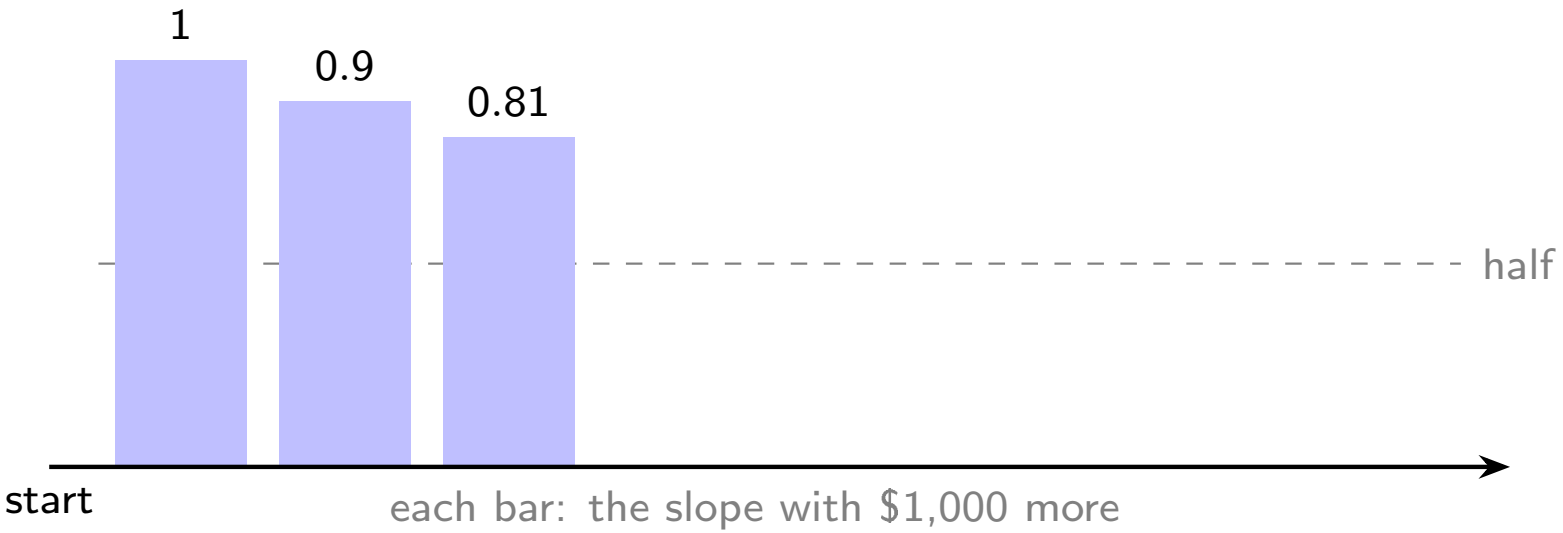
Start anywhere: call the slope there 1.

Seven Steps Halve the Slope



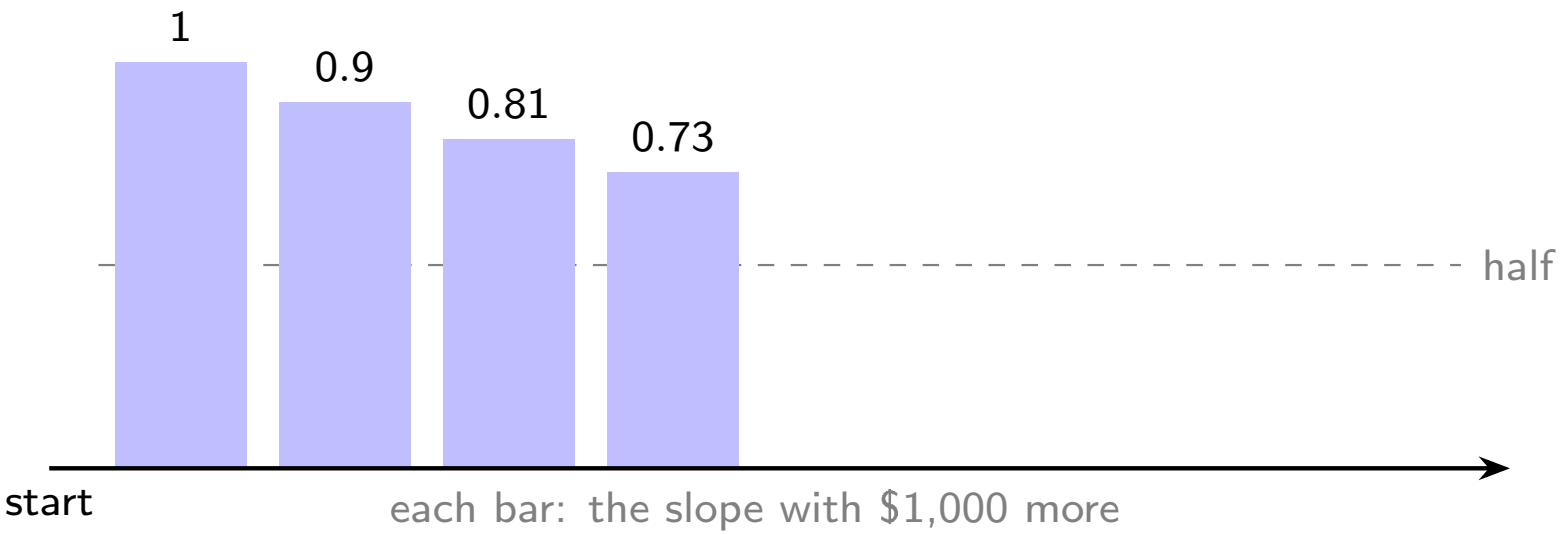
With \$1,000 more: $\times 0.9$.

Seven Steps Halve the Slope



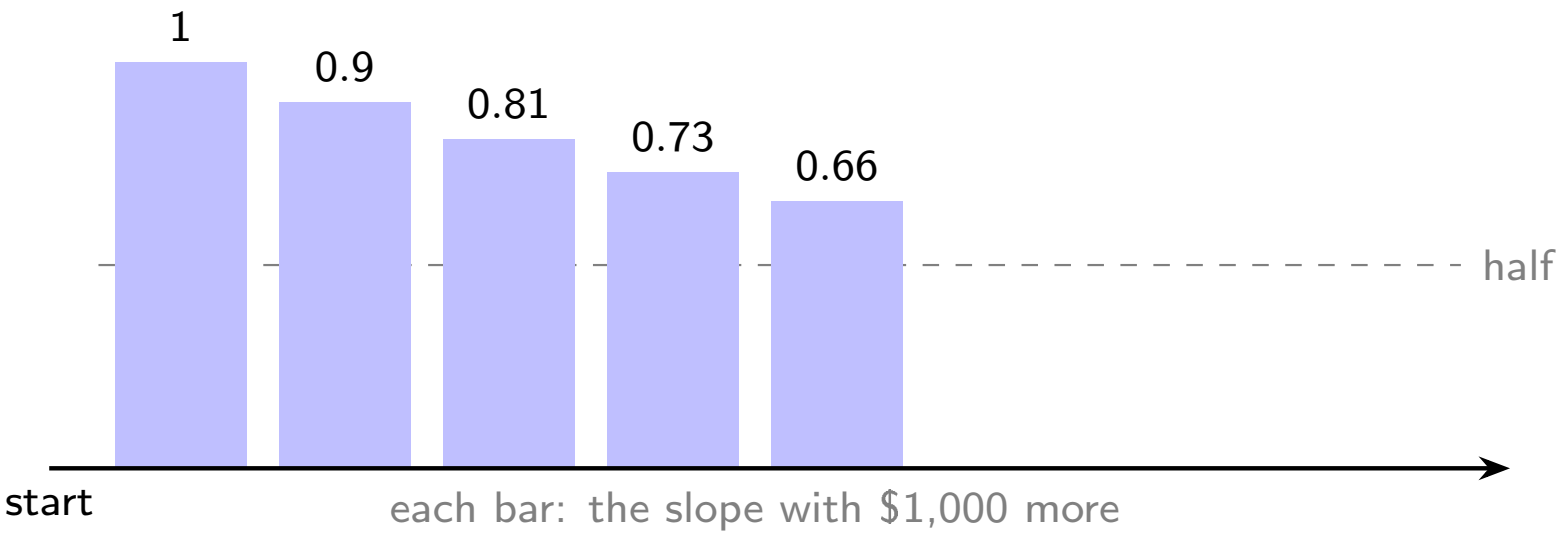
With \$2,000 more: $\times 0.9$ again, $0.9^2 = 0.81$.

Seven Steps Halve the Slope



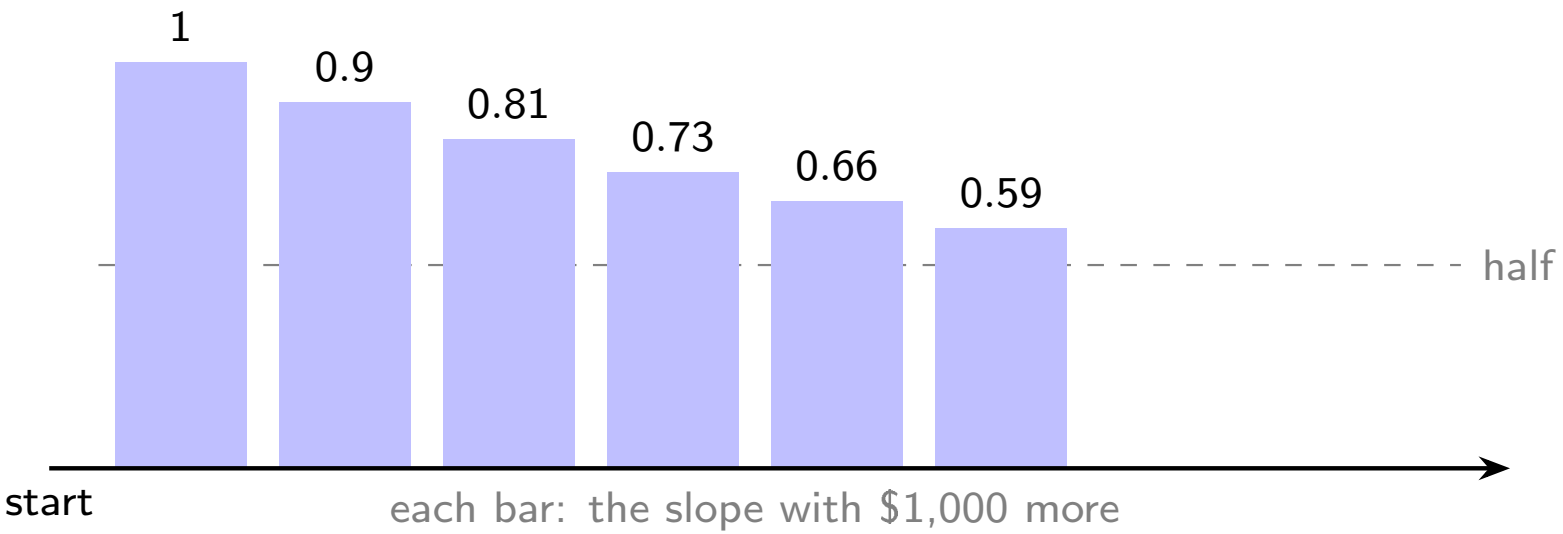
And again, with every $1,000 more.

Seven Steps Halve the Slope



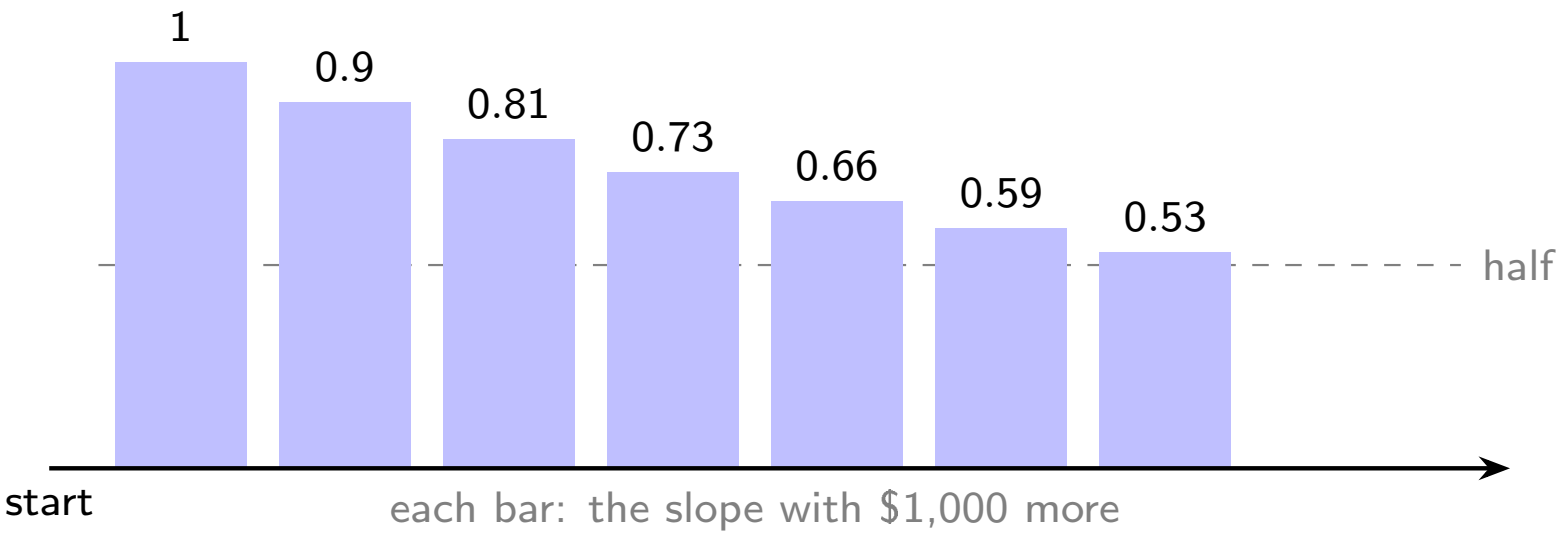
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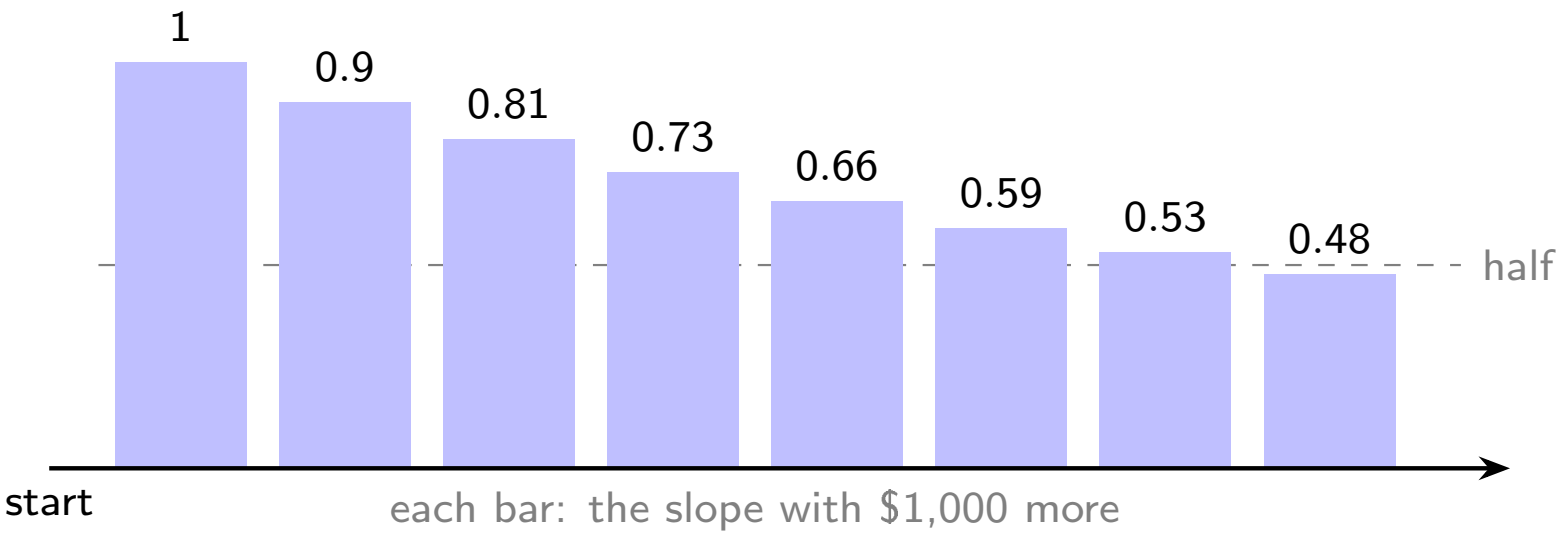
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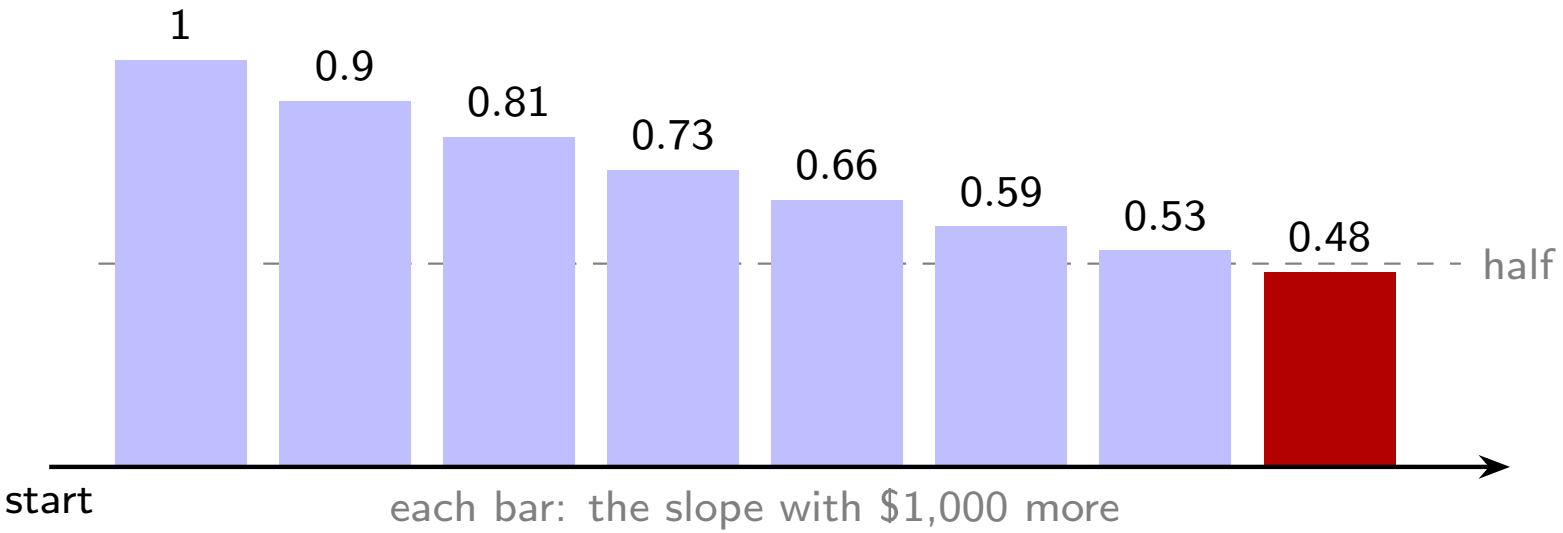
And again, with every $1,000 more.

Seven Steps Halve the Slope



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Seven Steps Halve the Slope



After seven steps, with \$7,000 more: $0.9^7 \approx 0.48$, less than half.
So with every \$7,000 more, the slope at least halves.

Halving, Again and Again



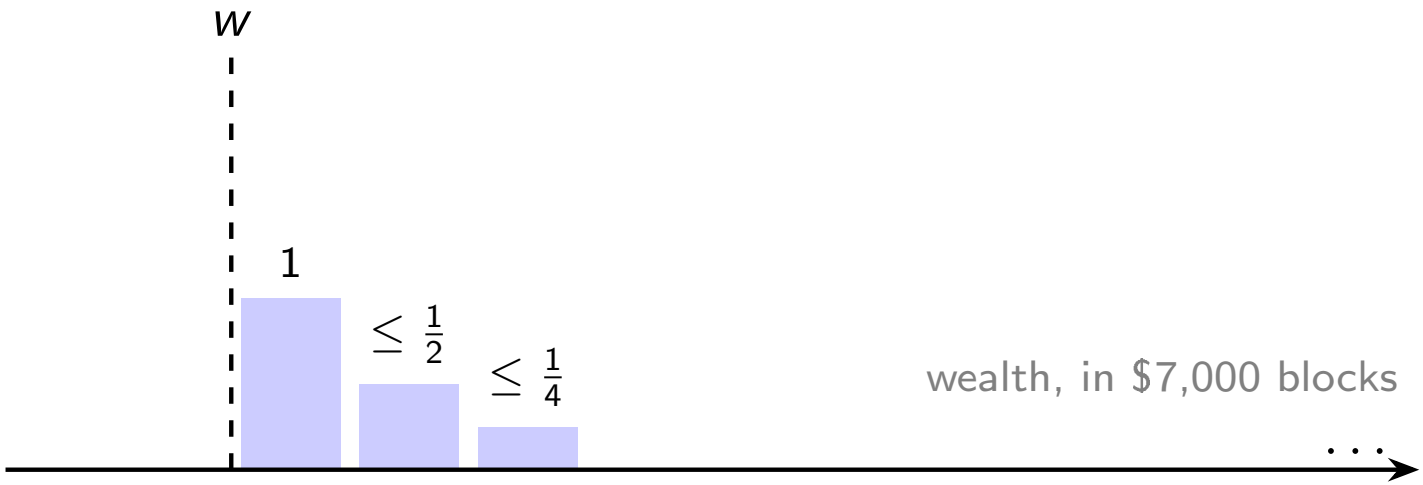
Bar height: what each \$7,000 block is worth, compared with the \$7,000 just above w. Call that one 1.

Halving, Again and Again



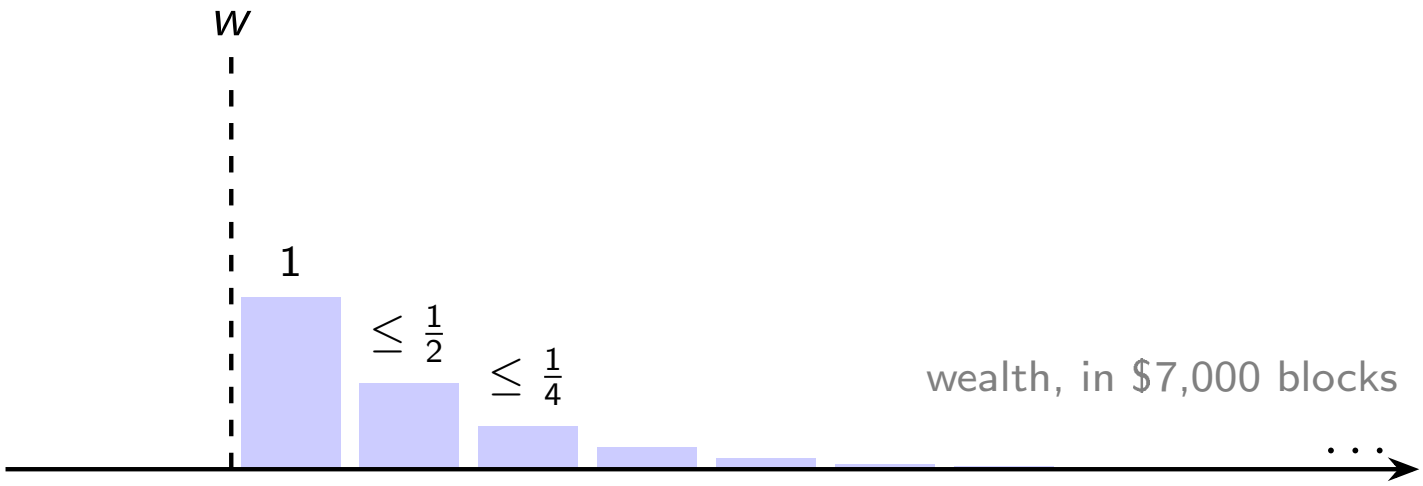
Every dollar in the next \$7,000 up is worth less than half the dollar \$7,000 below it.
So that block is worth at most $\frac{1}{2}$.

Halving, Again and Again



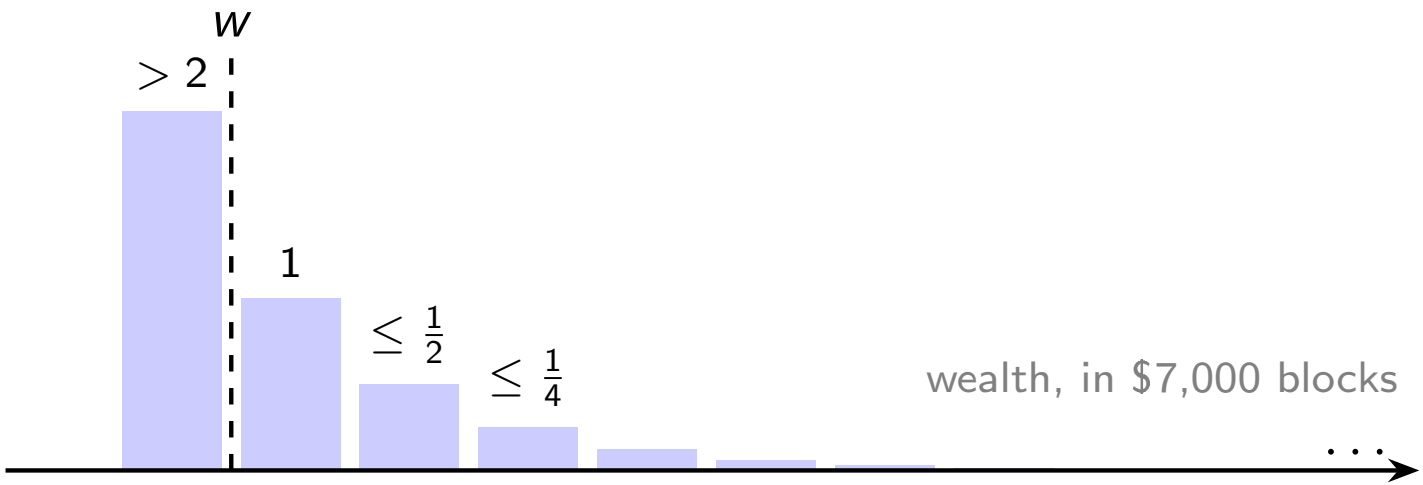
The next: at most $\frac{1}{4}$.

Halving, Again and Again



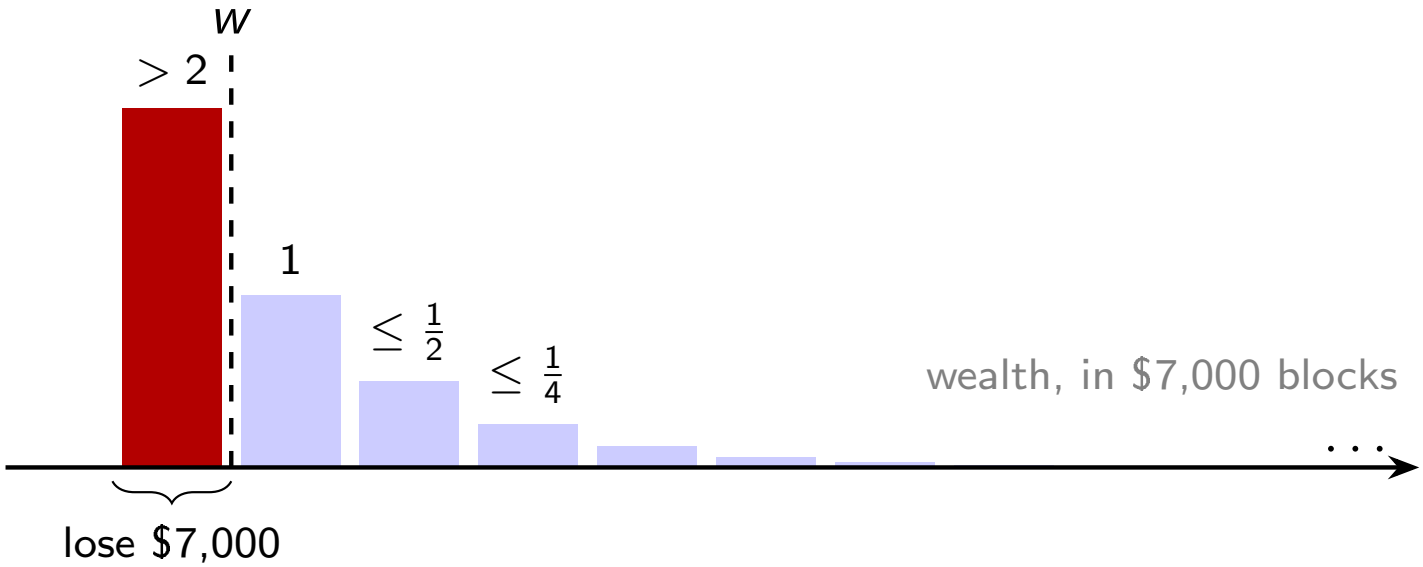
And so on, all the way up.

Halving, Again and Again



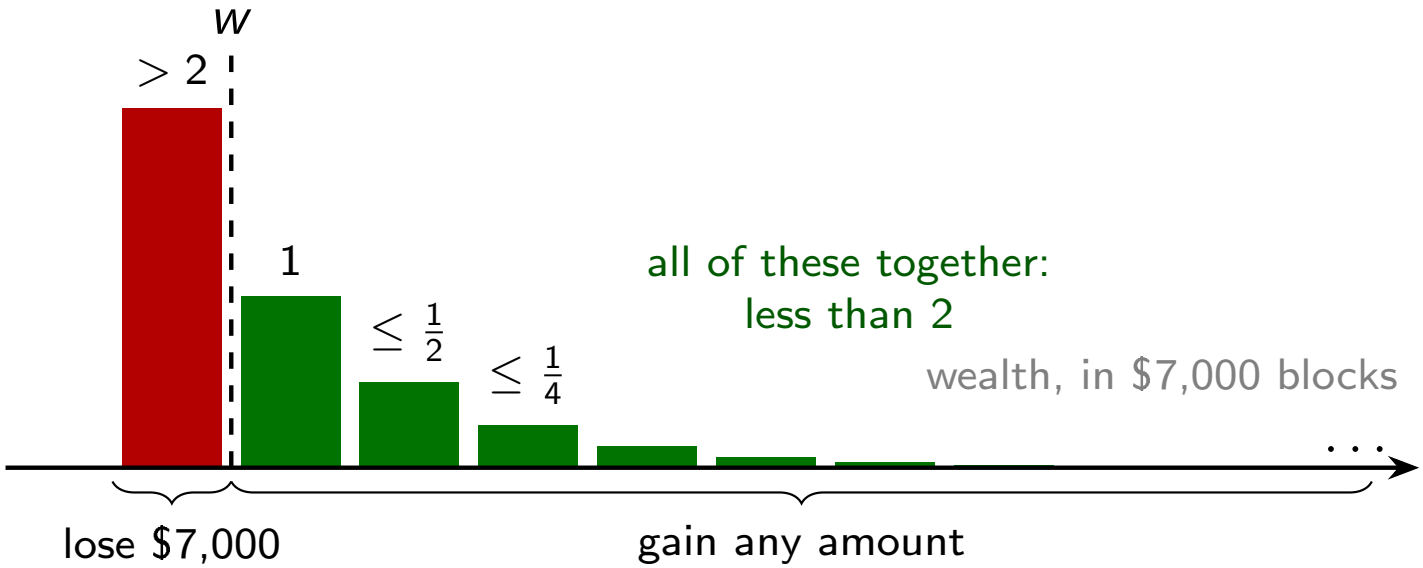
The other way: every dollar in the $7,000 just below w is worth more than
double the dollar $7,000 above it. So that block is worth more than 2.

Halving, Again and Again



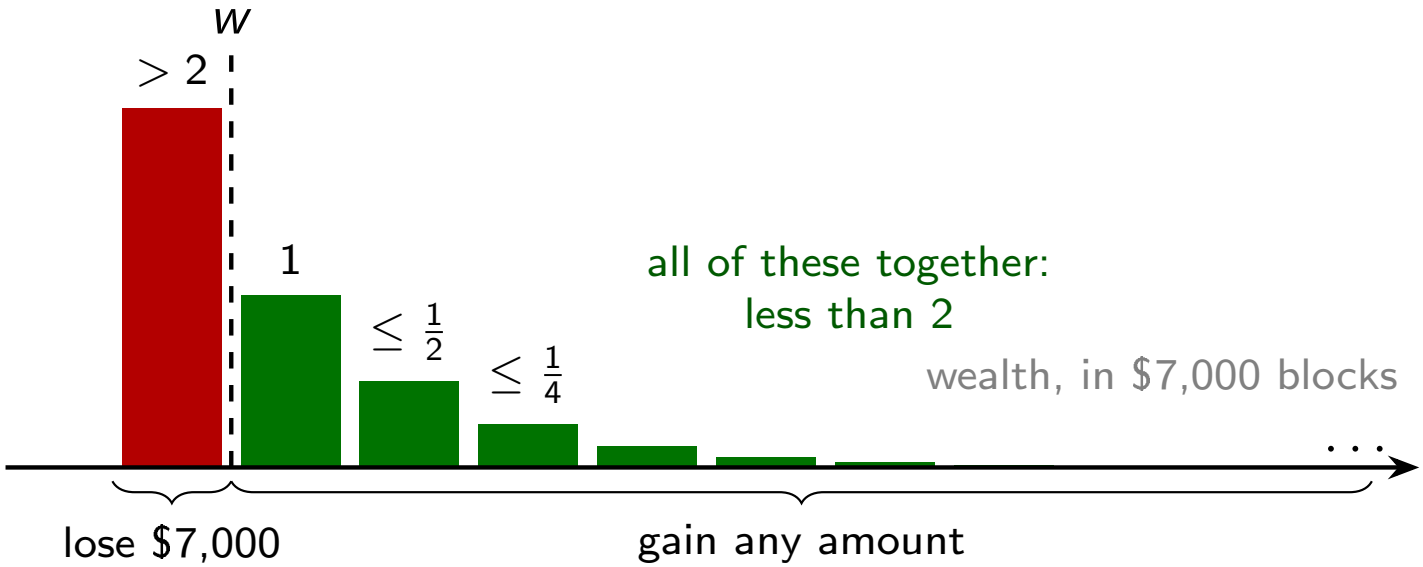
Lose $7,000: worth more than 2.

Halving, Again and Again



Gain any amount, however large: worth at most $1 + \frac{1}{2} + \frac{1}{4} + \ldots$, which stays below 2.

Halving, Again and Again



So they turn down 50–50 lose \$7,000 / gain G, however large G is: even $G = \$1{,}000{,}000{,}000$, an expected gain of \$499,996,500.

Calibration, More Generally

- Our step function is just one example.

Calibration, More Generally

- Our step function is just one example.
- Rabin (2000) showed that it works for every concave utility function that turns down 50–50 lose \$1,000 / gain \$1,100 at every wealth level.

Risk and Distribution

- Last week: states of nature play the role that people play.

Risk	Distribution

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Risk	Distribution
Equally likely states of nature	People

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Risk	Distribution
Equally likely states of nature	People
Utility of money	Wellbeing from money, or priority weighting
Turn down lose \$1,000 / gain \$1,100	Prefer (w, w) to $(w - 1{,}000,\ w + 1{,}100)$

So we can run the same argument twice more:
for a utilitarian about money, and for a prioritarian.

The Same Problem for Money (1)

- Two people, equally well off.

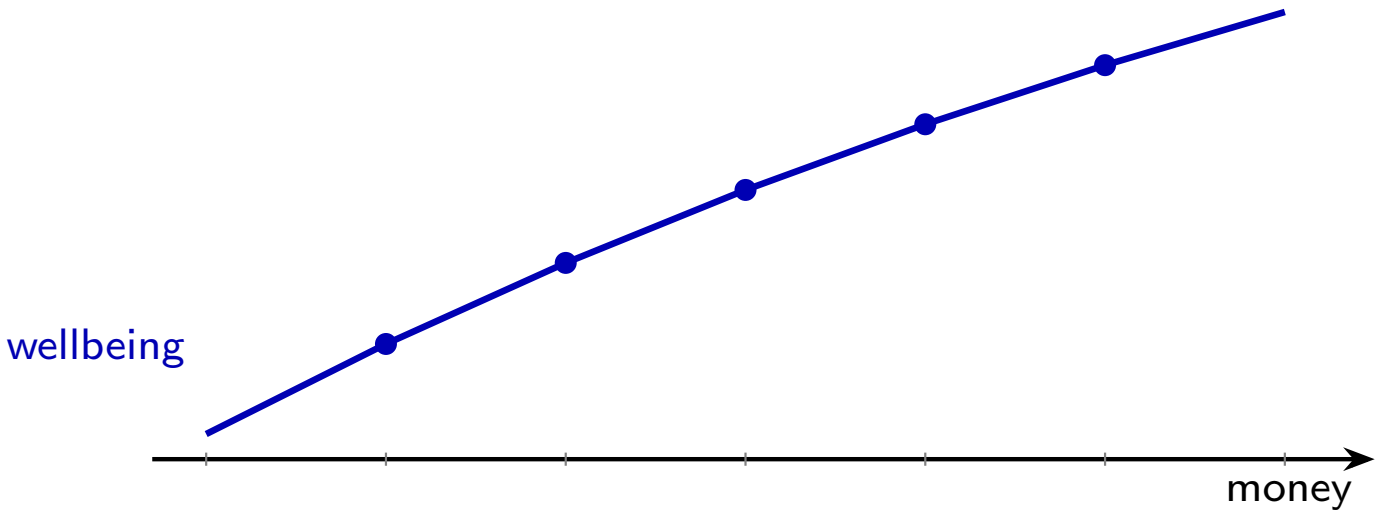
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- Two people, equally well off.
- Is it better to take $1,000 from one and give $1,100 to the other?
- Is it better to take $7,000 from one and give $1,000,000,000 to the other?

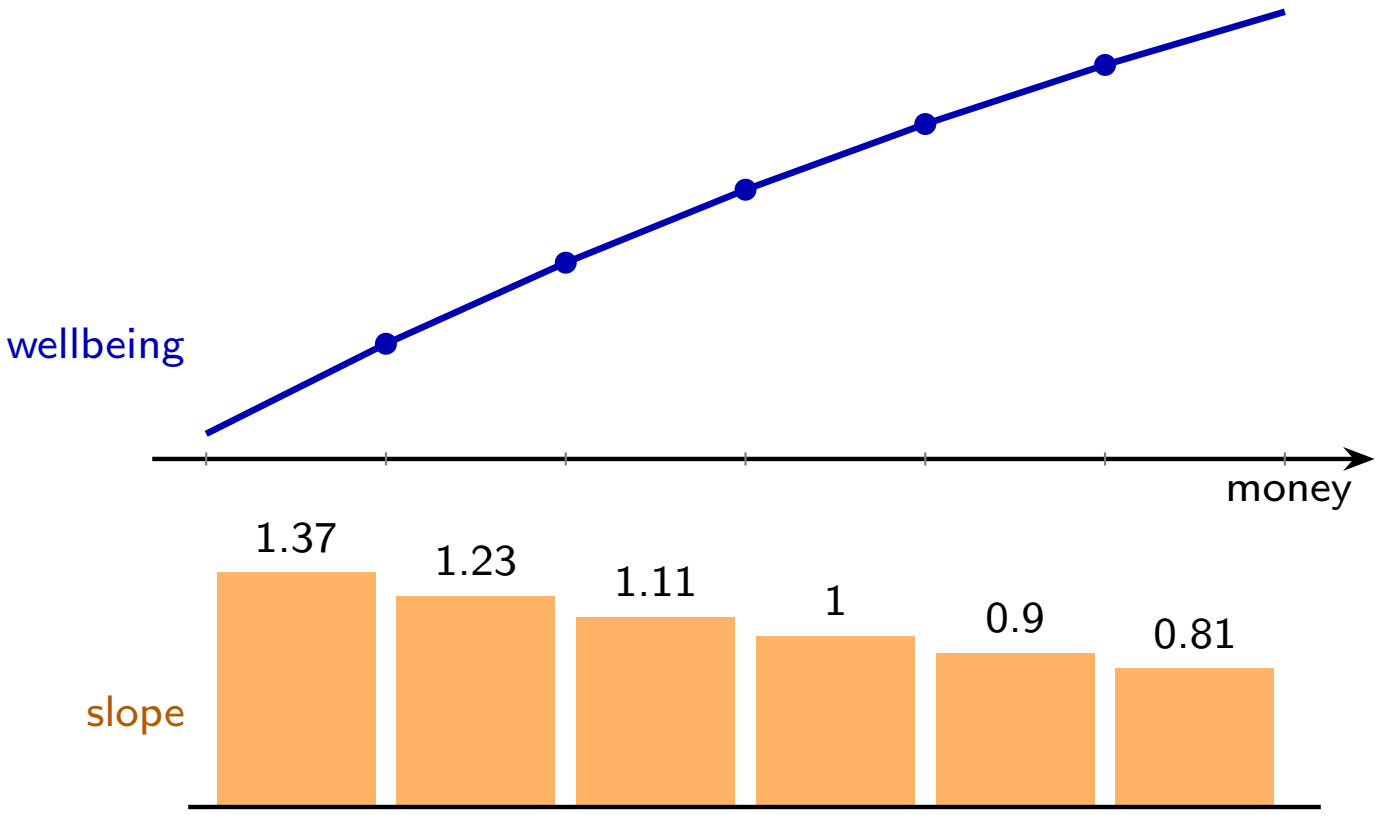
Wellbeing From Money in Straight Pieces



A utilitarian who never takes \$1,000 from one person to give \$1,100 to another equally well off.

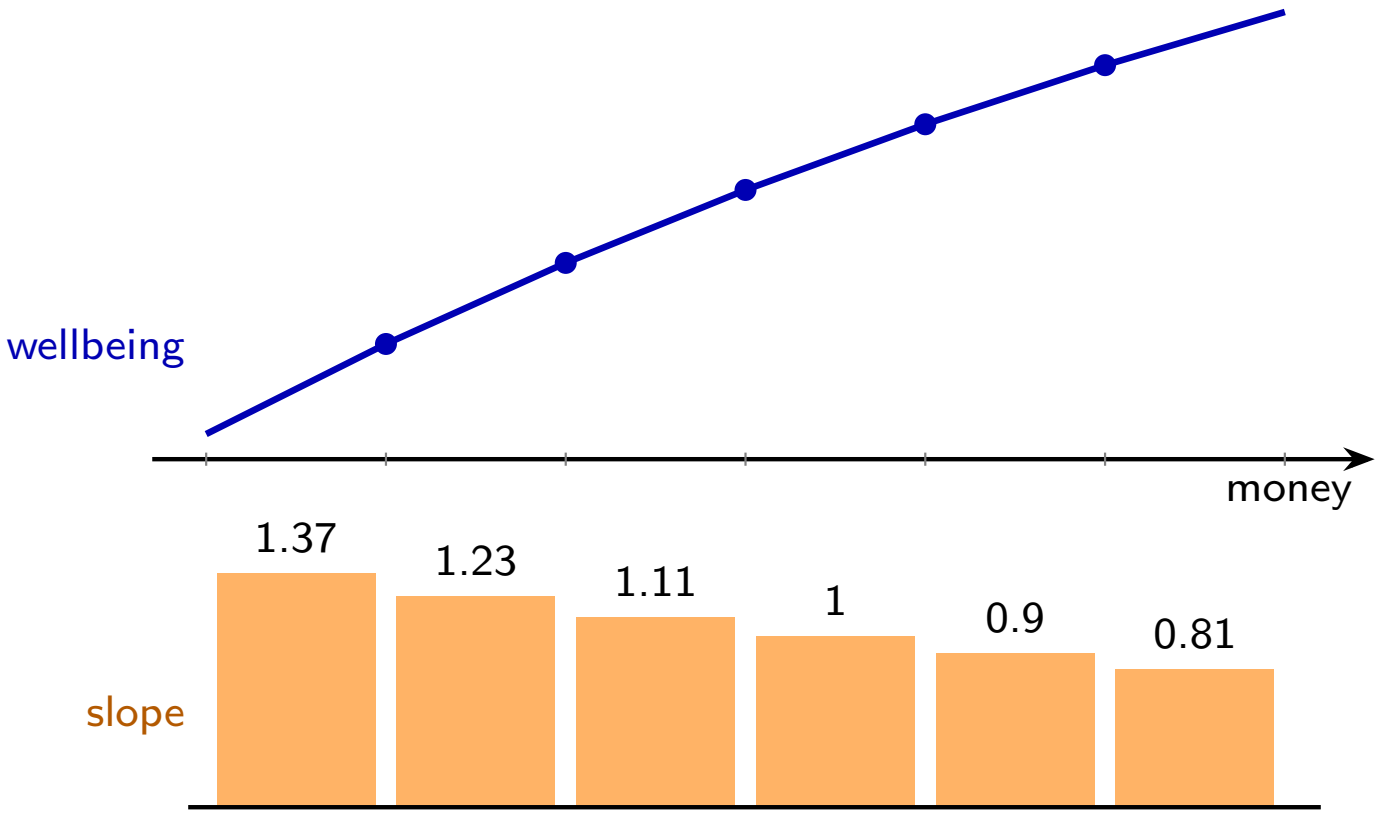
One way wellbeing could depend on money: straight \$1,000 pieces.

Wellbeing From Money in Straight Pieces



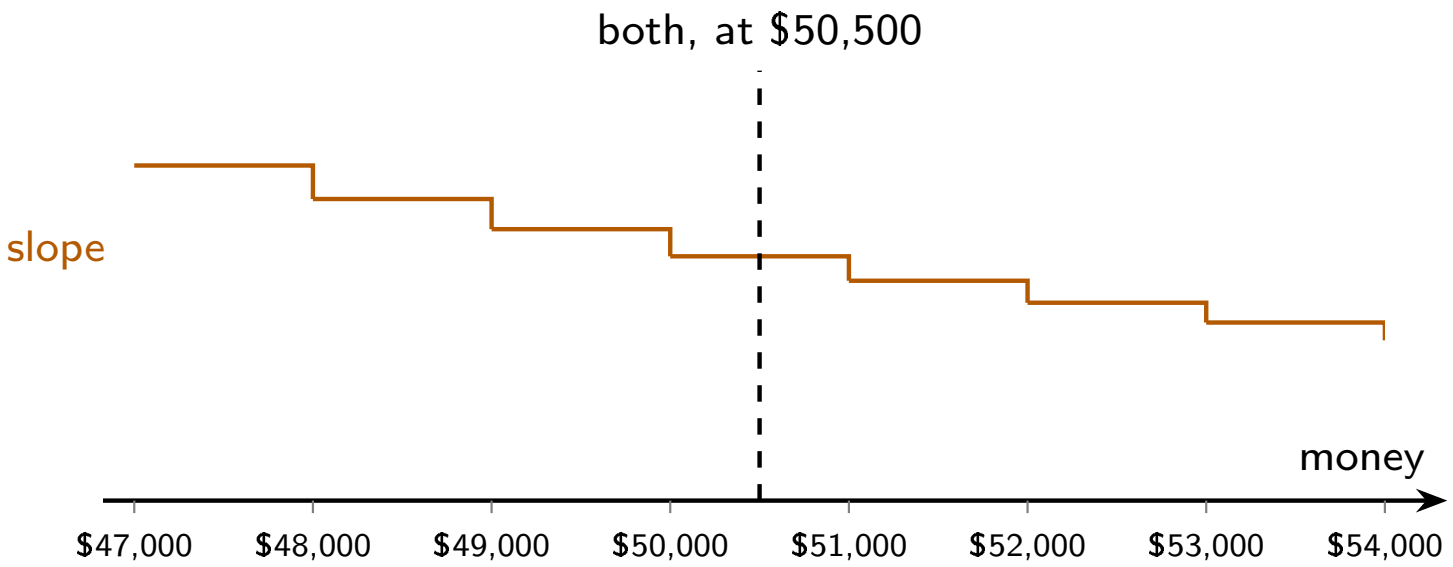
Below: the slope of each piece.
The slope is how much wellbeing each extra dollar adds at that point.

Wellbeing From Money in Straight Pieces



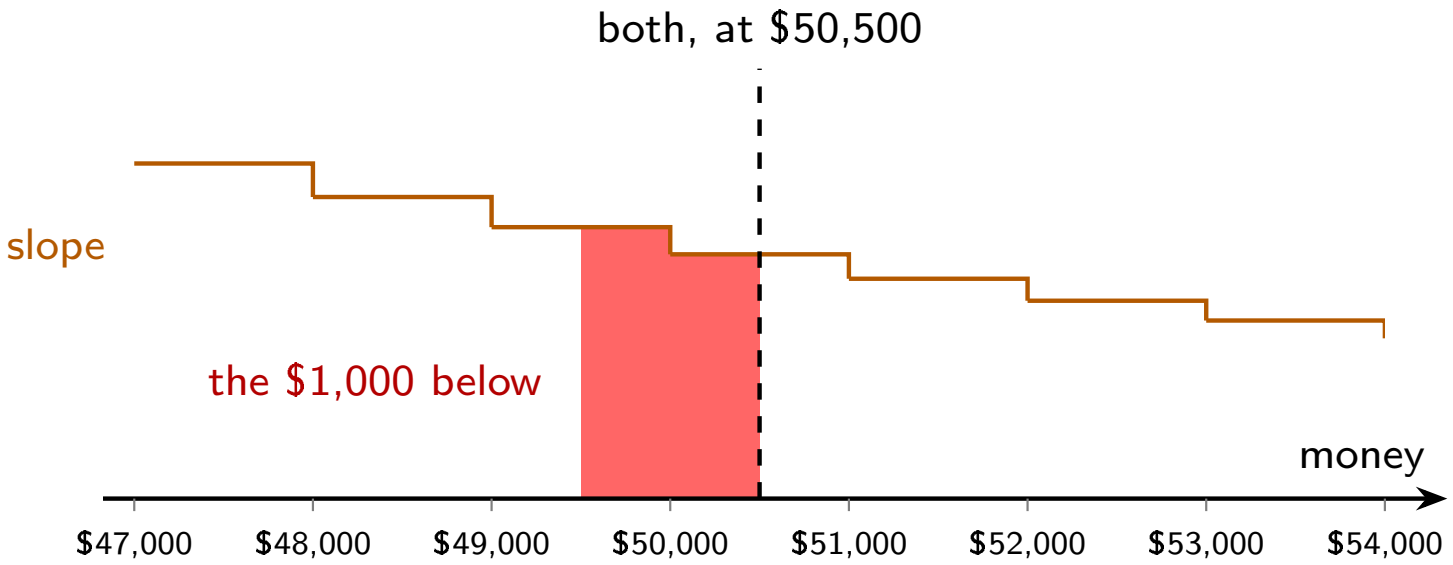
Every \$1,000, the slope is multiplied by 0.9.

Diminishing Marginal Wellbeing



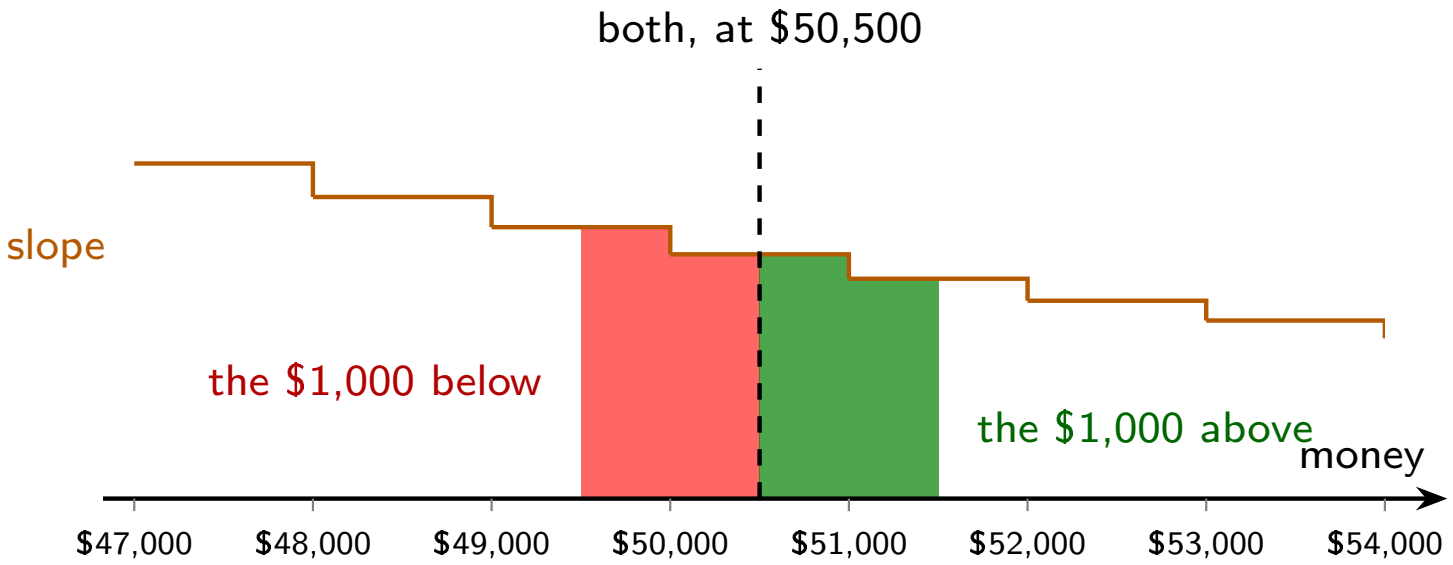
Two people, equally well off, each with $50,500: in the middle of a piece.

Diminishing Marginal Wellbeing



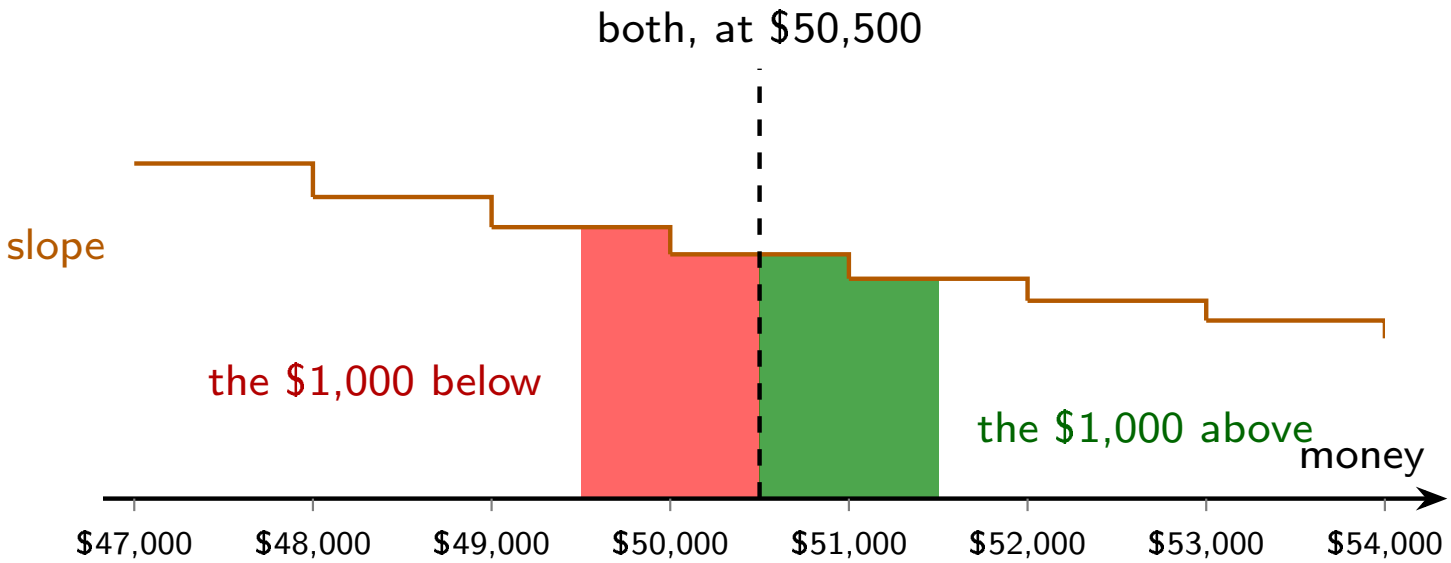
Red: the wellbeing one person loses if we take $1,000 from them.

Diminishing Marginal Wellbeing



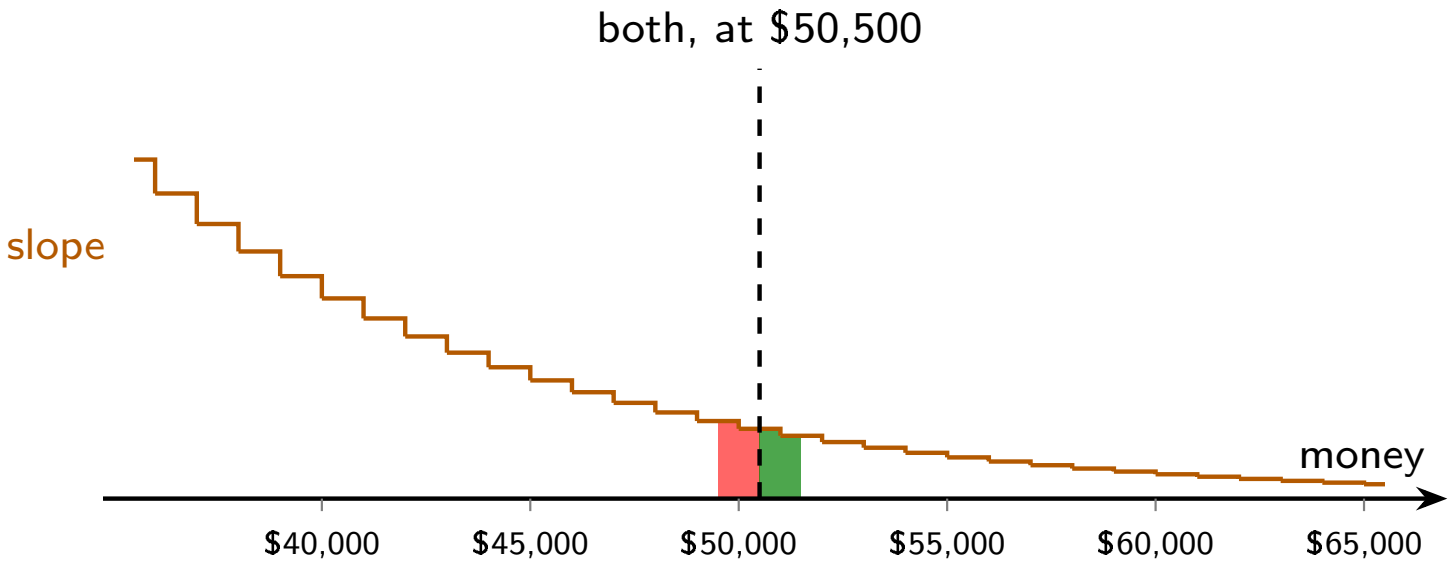
Green: the wellbeing the other gains from $1,000 more.
It's the red area moved up $1,000, where every slope is 0.9 as big.
So it's worth 0.9 of the red.

Diminishing Marginal Wellbeing



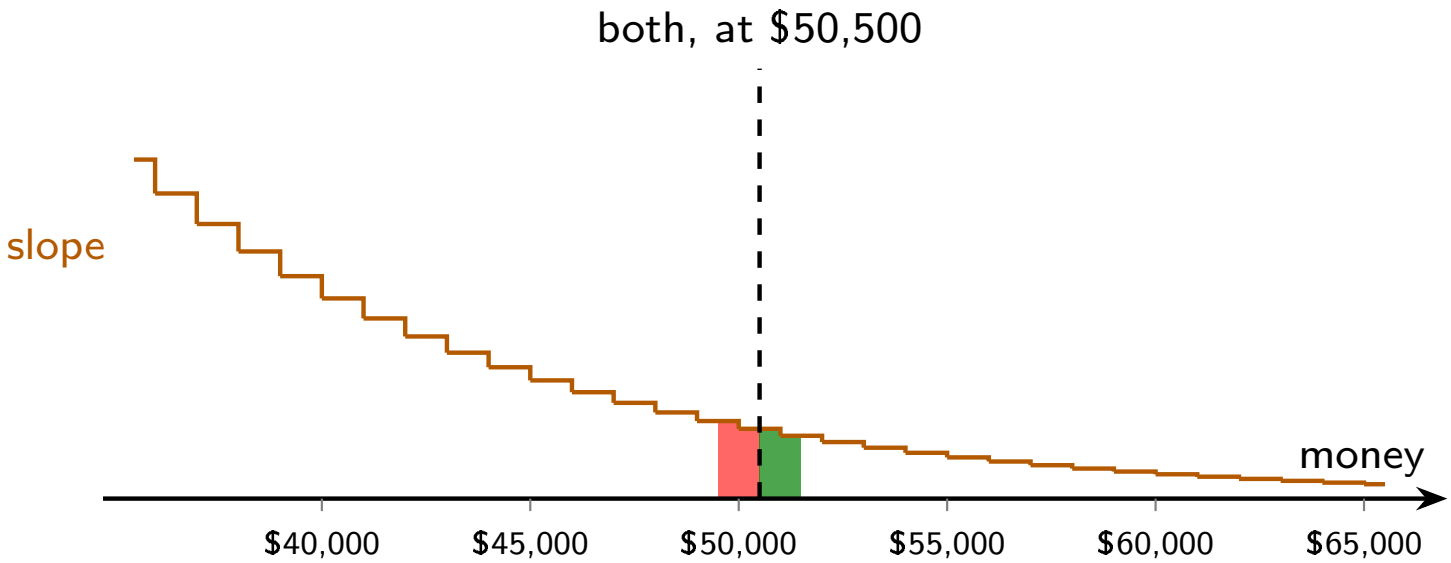
So \$1,100 more for one adds at most $0.9 + 0.09 = 0.99$ of what \$1,000 less costs the other: the drop-off needed for the utilitarian never to take \$1,000 from one to give \$1,100 to the other.

Diminishing Marginal Wellbeing



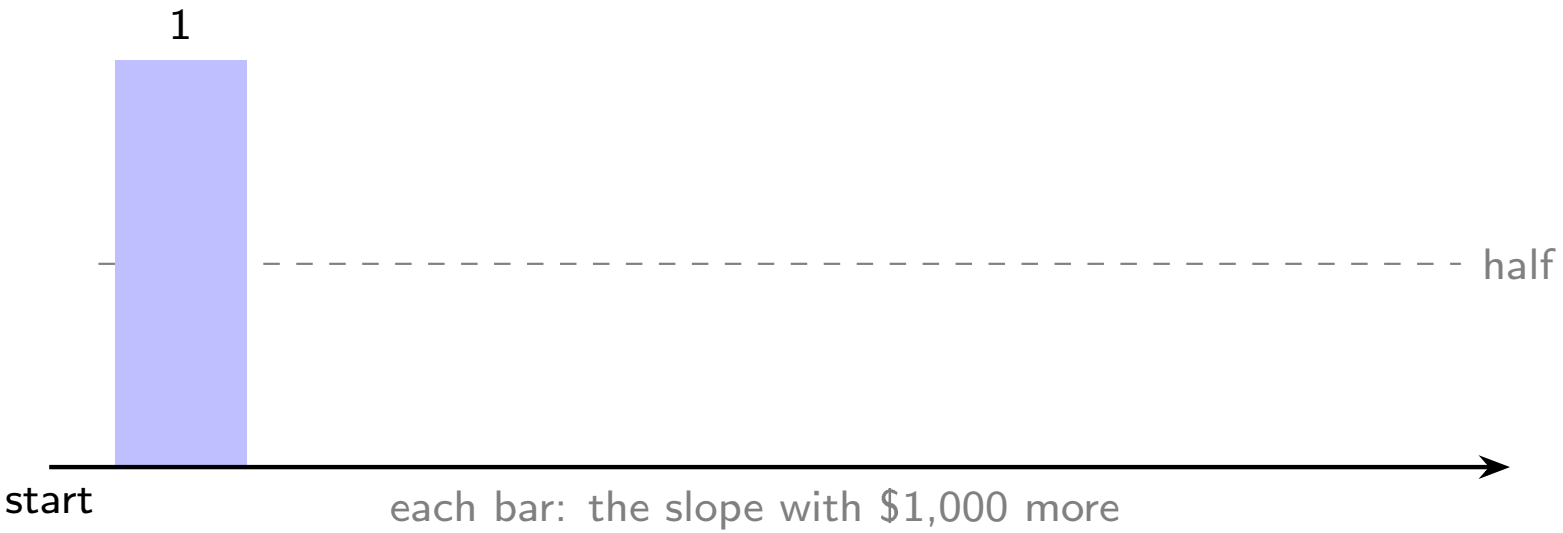
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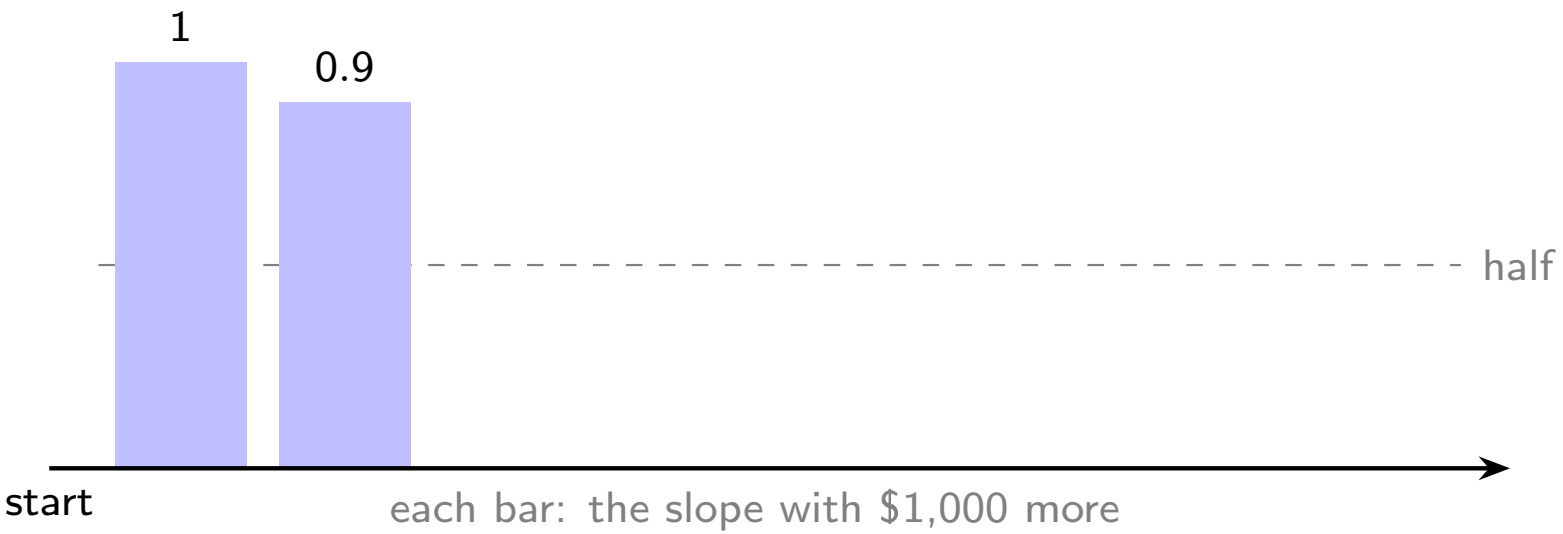
Zoomed out, from about $35,000 to $65,000:
the less money, the higher and higher the slope.

Seven Steps Halve the Wellbeing per Dollar



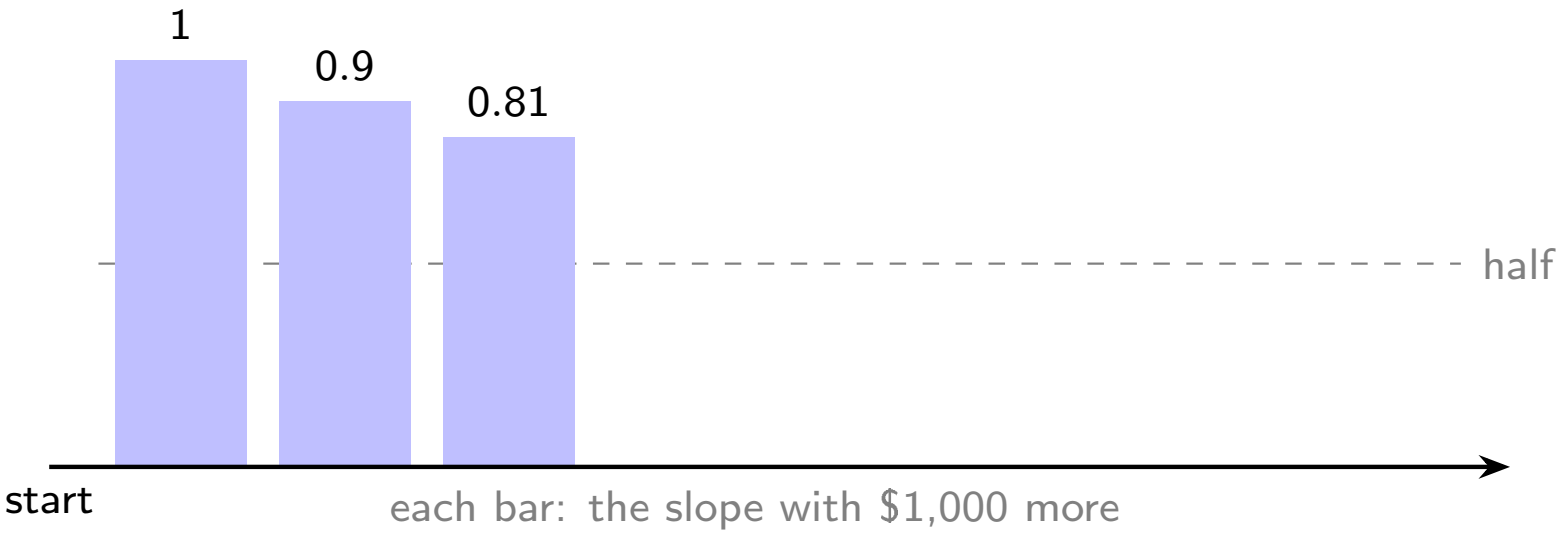
Start anywhere: call the slope there 1.

Seven Steps Halve the Wellbeing per Dollar



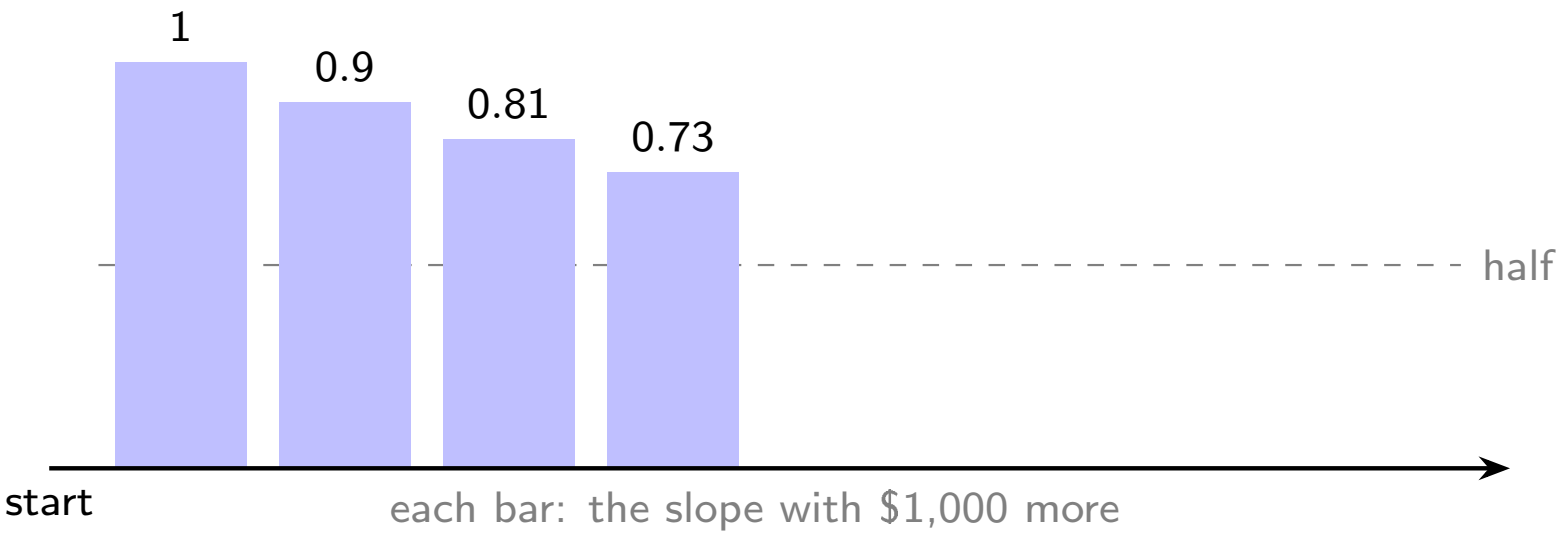
With \$1,000 more: $\times 0.9$.

Seven Steps Halve the Wellbeing per Dollar



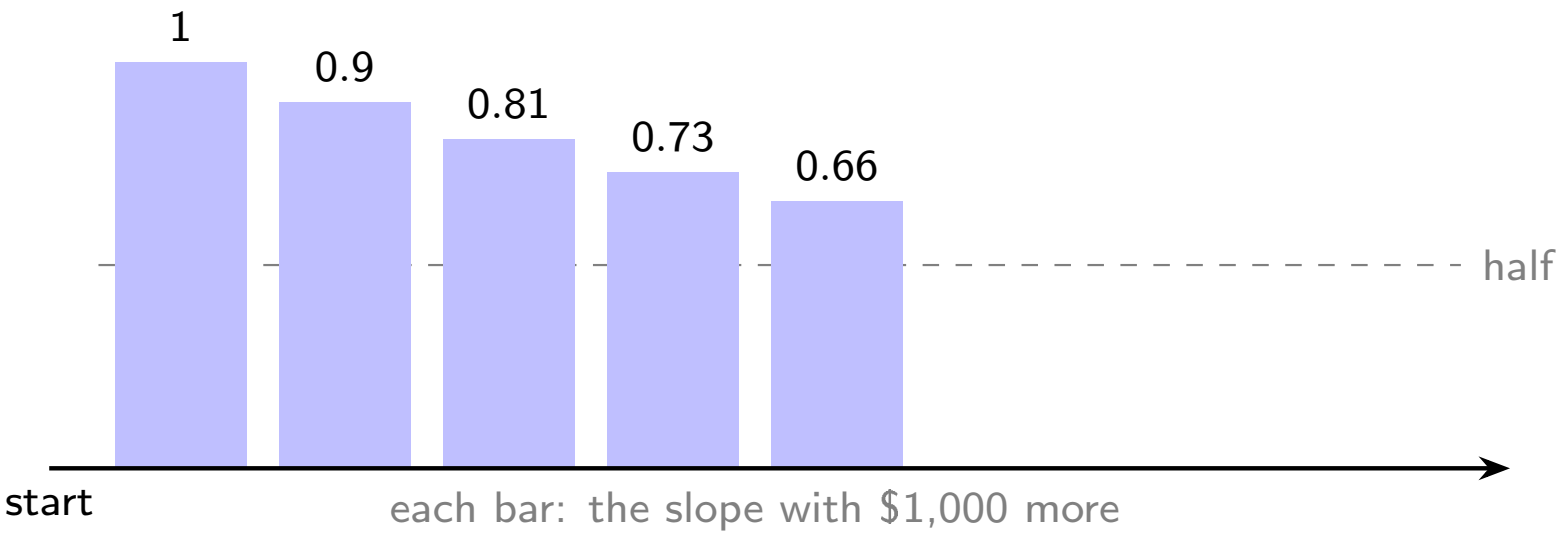
With \$2,000 more: $0.9^2 = 0.81$.

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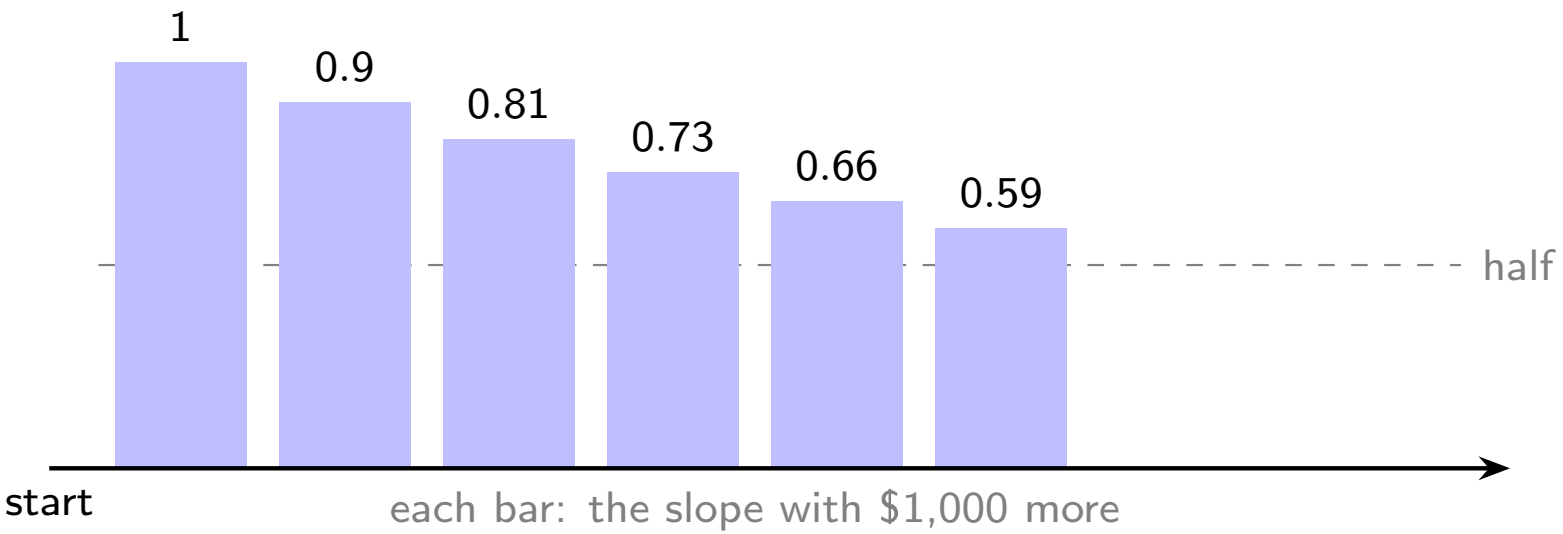
And again, with every $1,000 more.

Seven Steps Halve the Wellbeing per Dollar



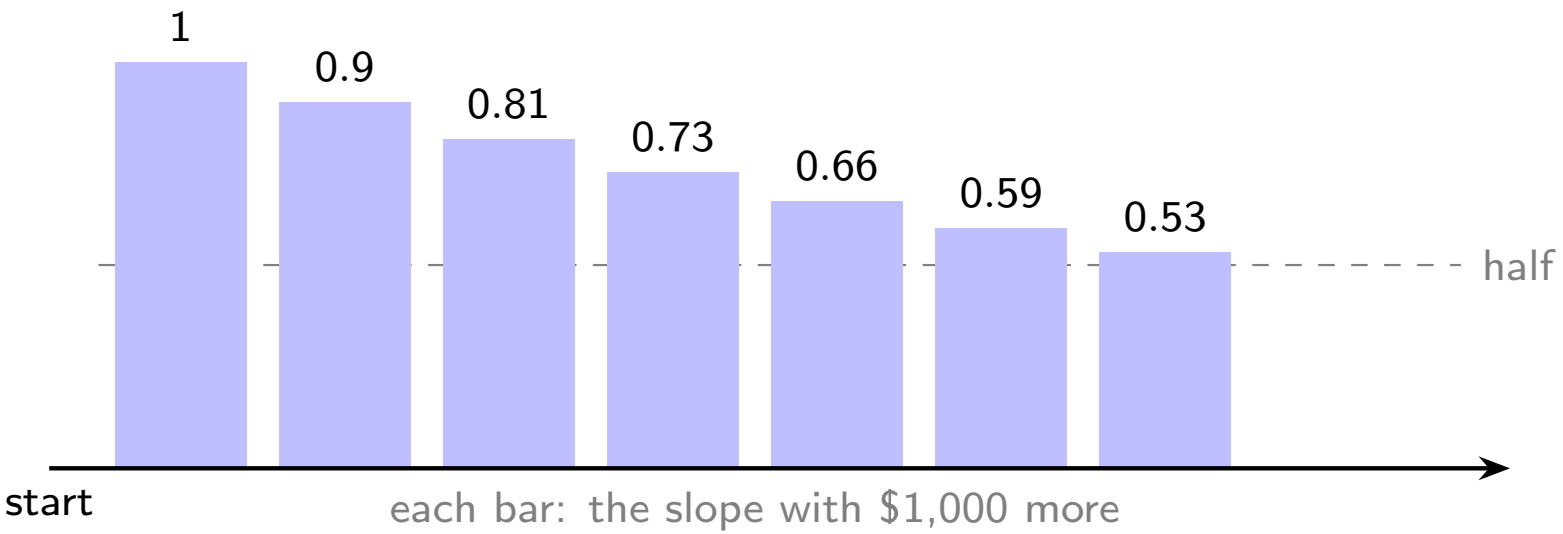
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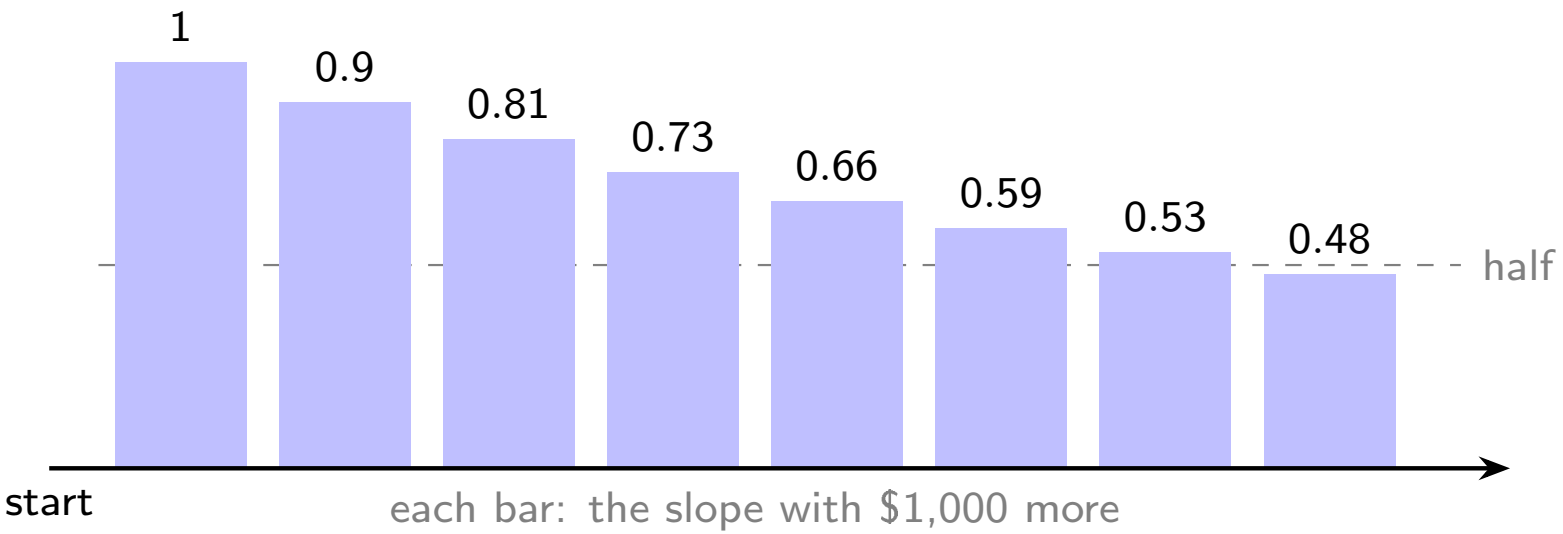
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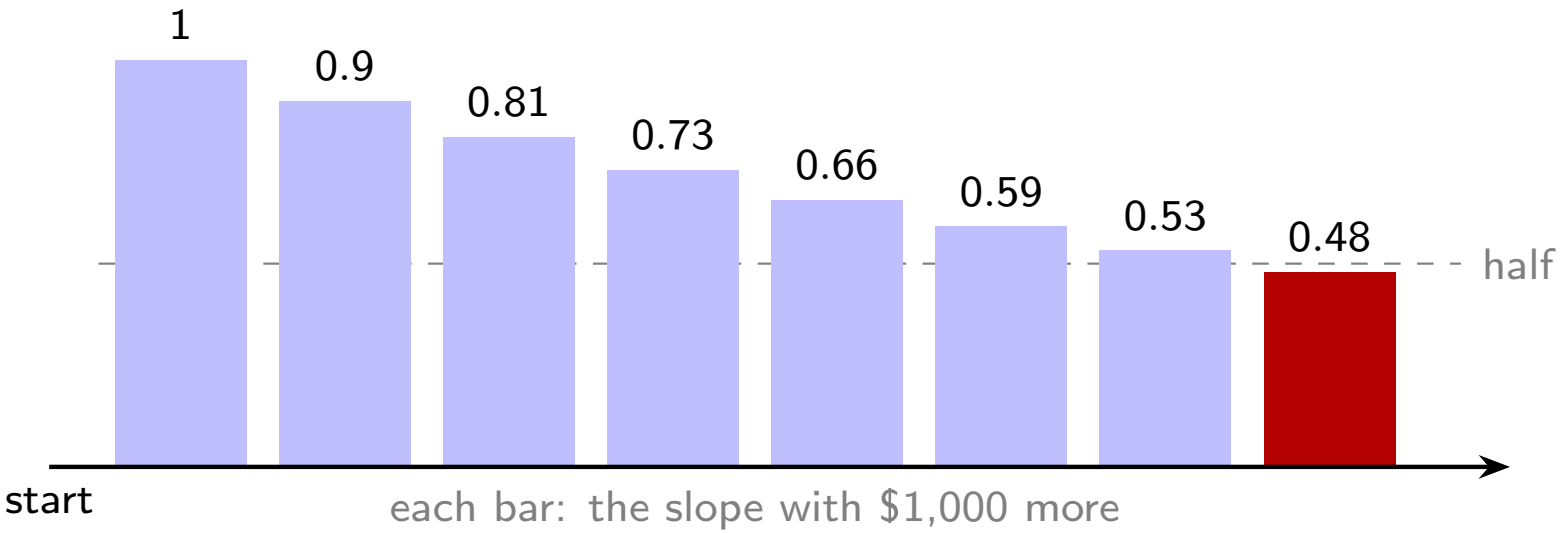
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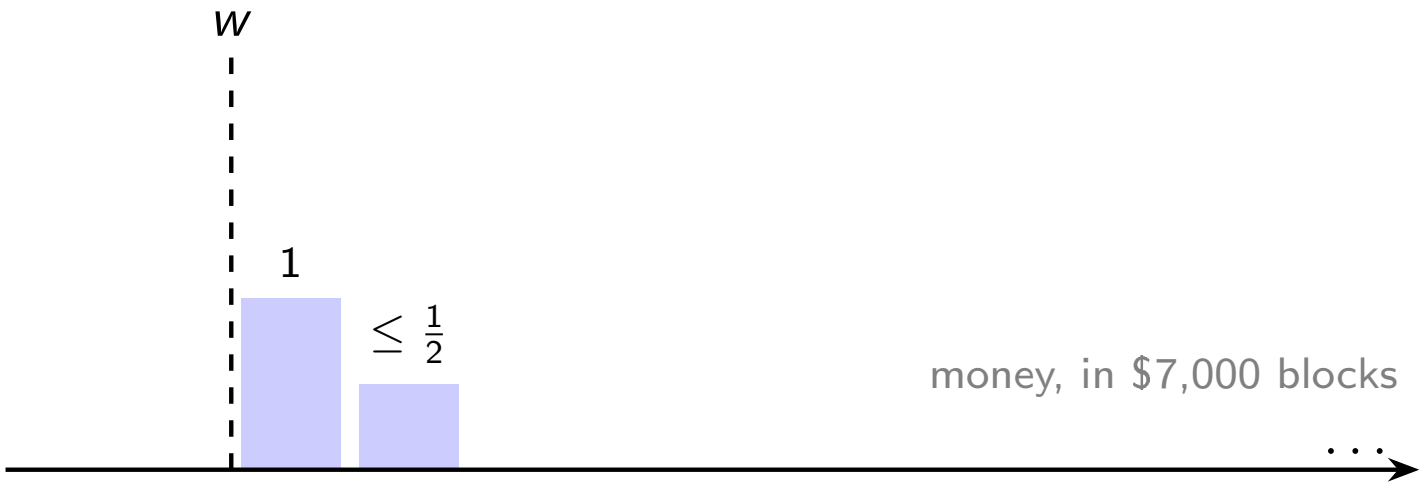
After seven steps, with \$7,000 more: $0.9^7 \approx 0.48$, less than half.
So with every \$7,000 more, the wellbeing per dollar at least halves.

Halving for Utilitarians



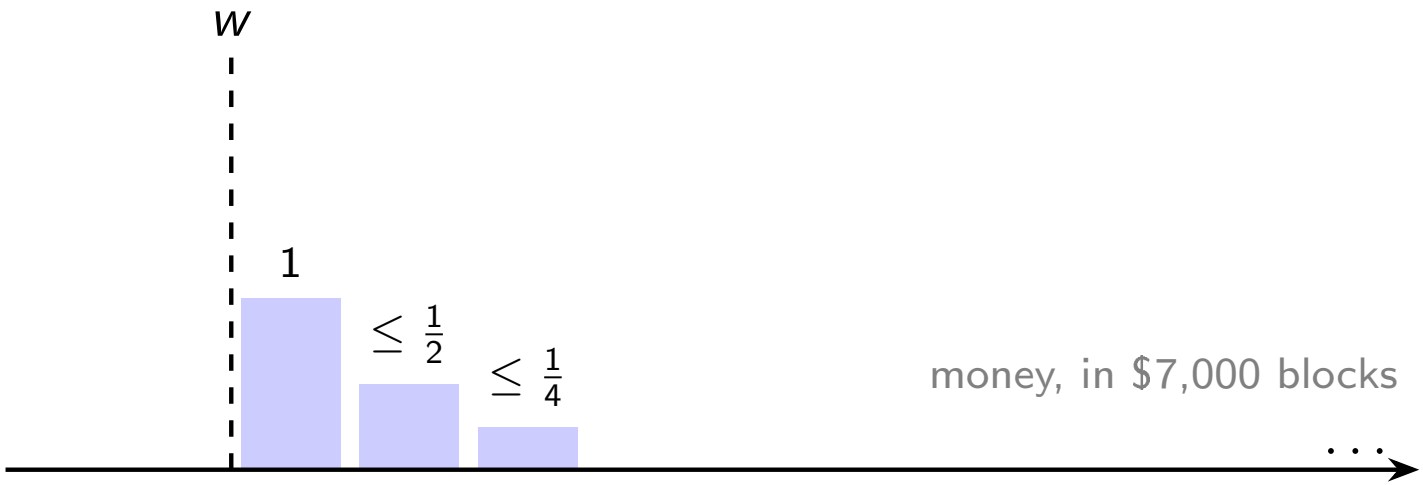
Bar height: the wellbeing each \$7,000 block adds, compared with the \$7,000 just above w. Call that one 1.

Halving for Utilitarians



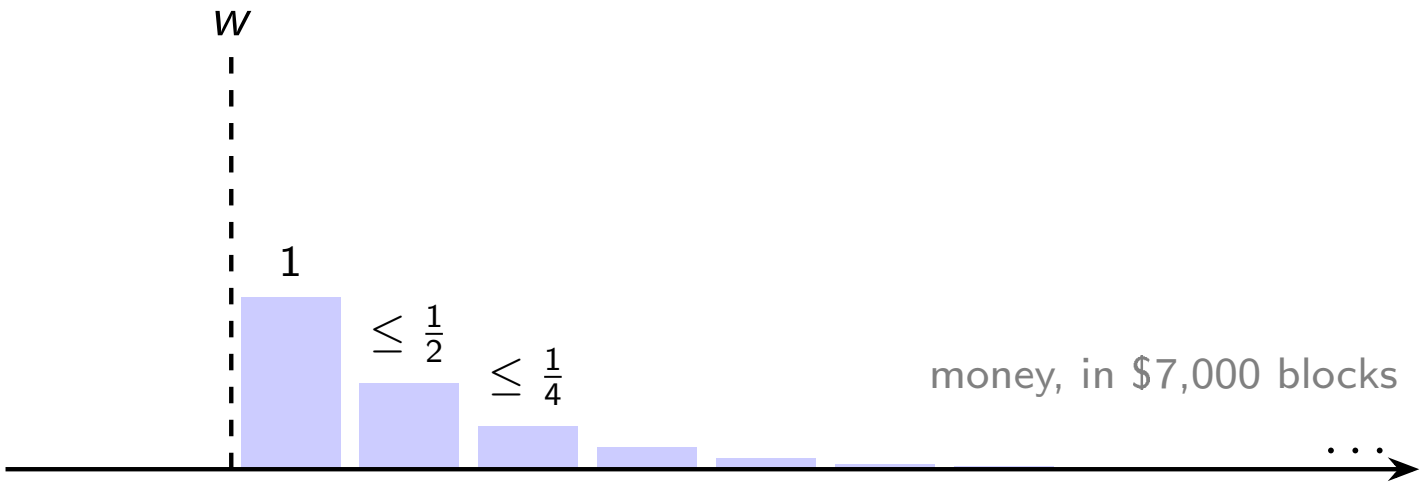
The next \$7,000 up: at most $\frac{1}{2}$.

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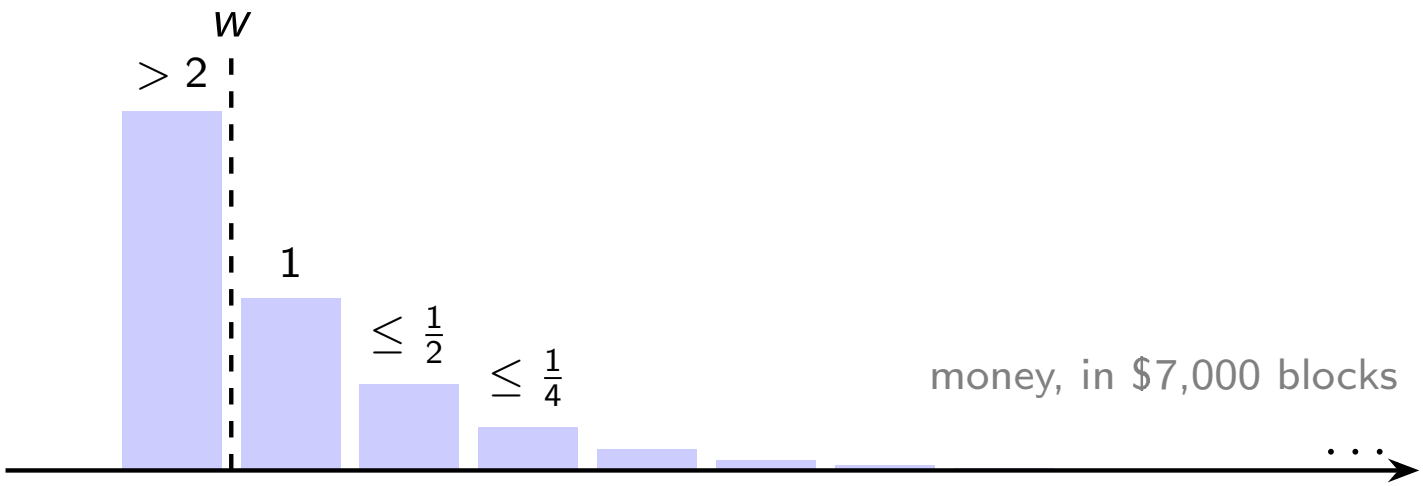
The next: at most $\frac{1}{4}$.

Halving for Utilitarians



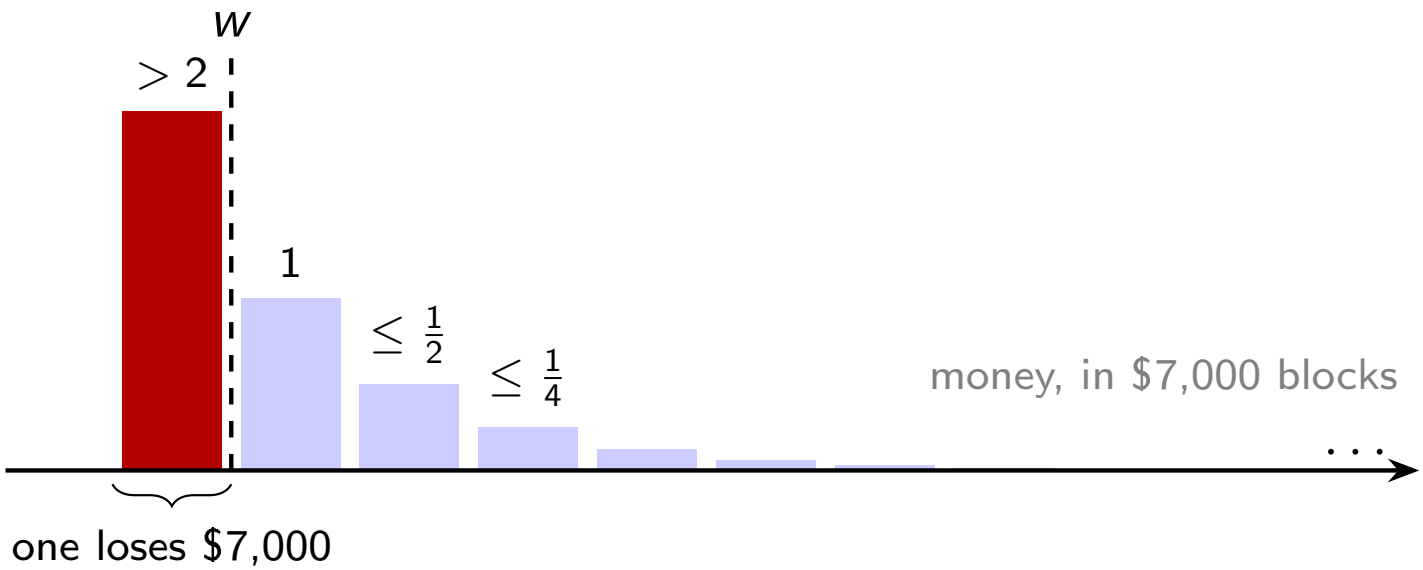
And so on, all the way up.

Halving for Utilitarians



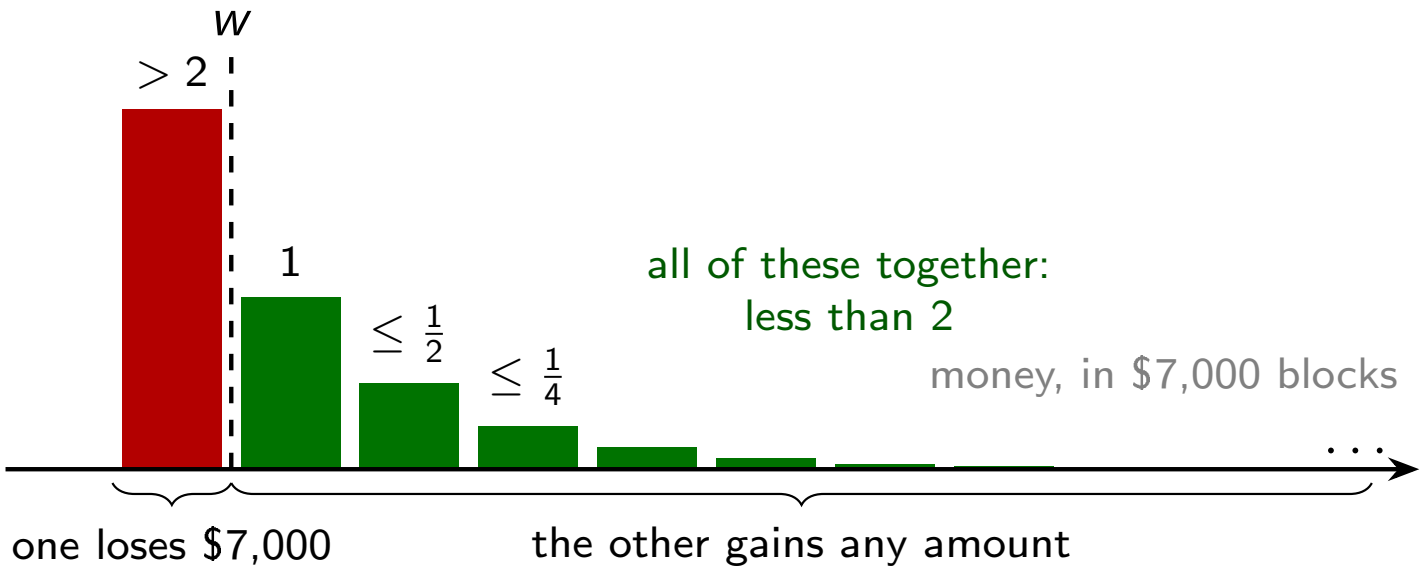
The other way: the \$7,000 just below w adds more than 2.

Halving for Utilitarians



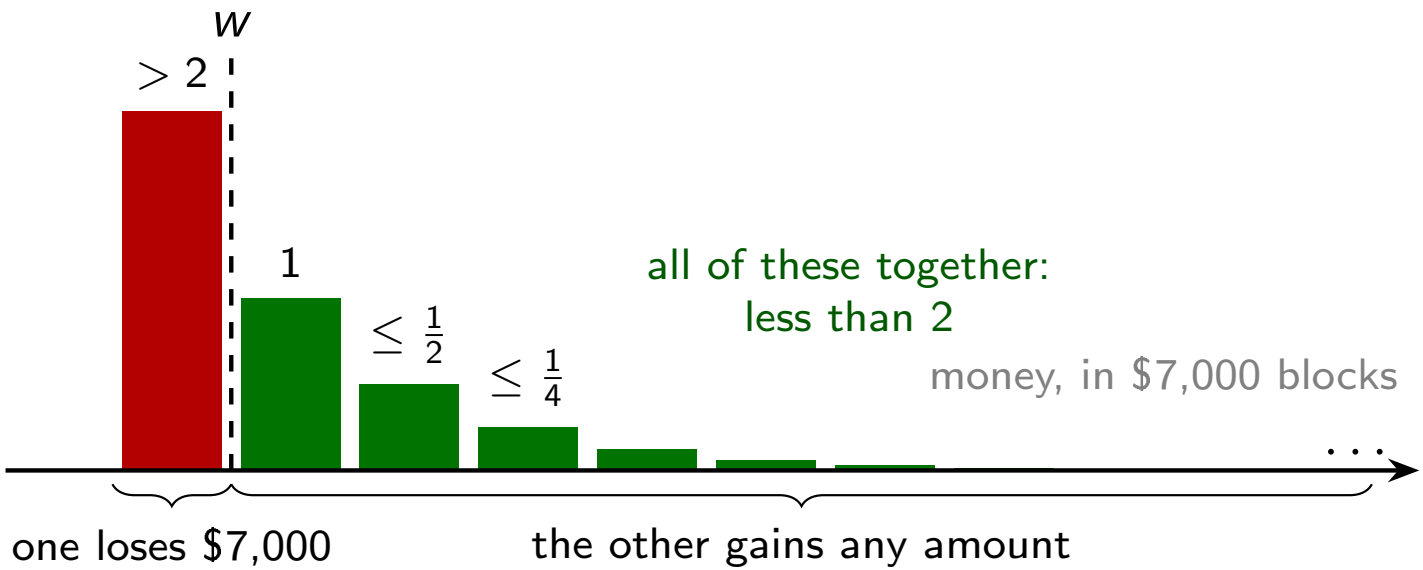
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The other gains any amount, however large: that adds at most $1 + \frac{1}{2} + \frac{1}{4} + \ldots$, which stays below 2.

Halving for Utilitarians



So the utilitarian won't take $7,000 from one to give G to the other, however large G is: not even $1,000,000,000.

What This Means for Utilitarians

Risk	Money, for a utilitarian
One person, two equally likely states	Two people, equally well off

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Every \$1,000, each dollar adds 0.9 as much wellbeing: the same picture as before.

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One person, two equally likely states	Two people, equally well off
Utility per dollar	Wellbeing per dollar
Turn down lose \$1,000 / gain \$1,100	Don't take \$1,000 from one to give \$1,100 to the other
Turn down lose \$7,000 / gain G	Don't take \$7,000 from one to give G to the other

However large G is.

So the utilitarian explanation of our equality-favouring judgements about money runs into the calibration problem too.

Priority Weights in Straight Pieces

- The priority weight: how much each extra unit of wellbeing adds to the priority-weighted sum, at that level.

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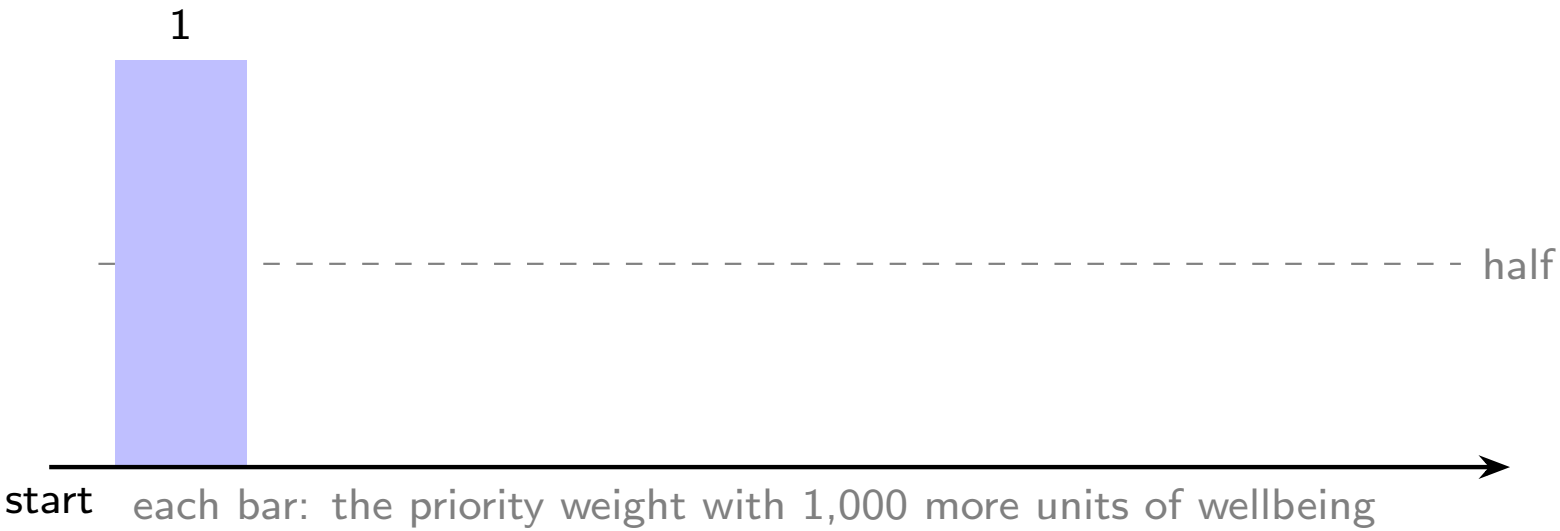
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Priority Weights in Straight Pieces

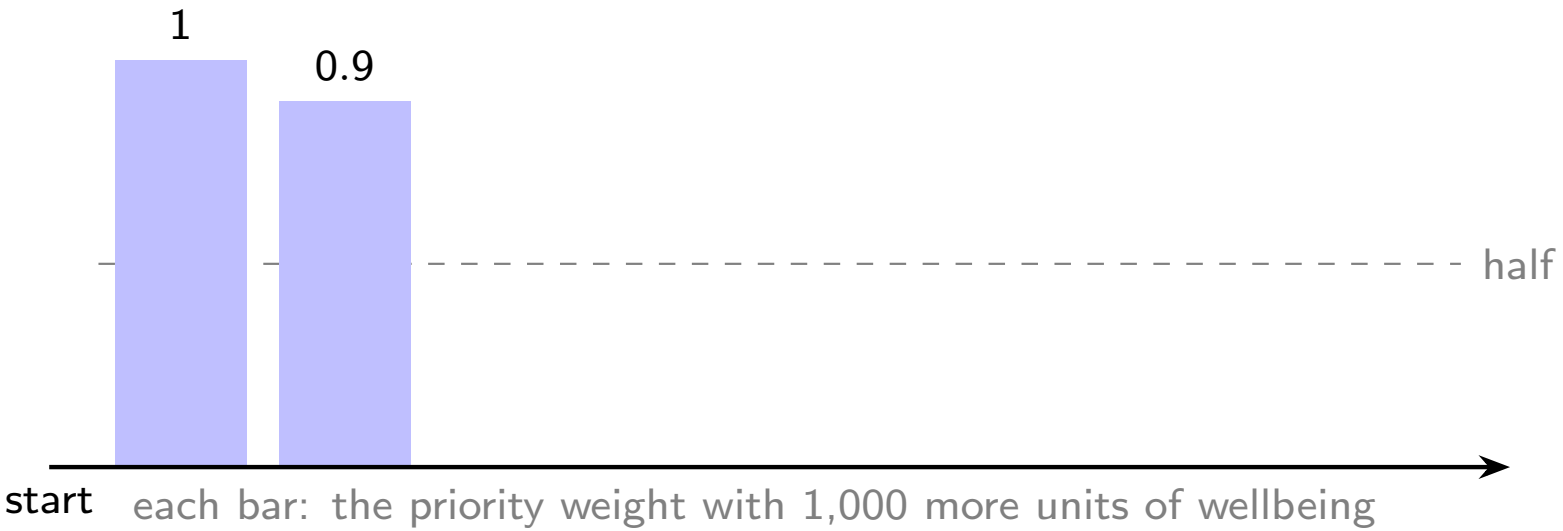
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- So this prioritarian prefers (w, w) to $(w - 1{,}000, w + 1{,}100)$, at every level w.

Seven Steps Halve the Priority Weight



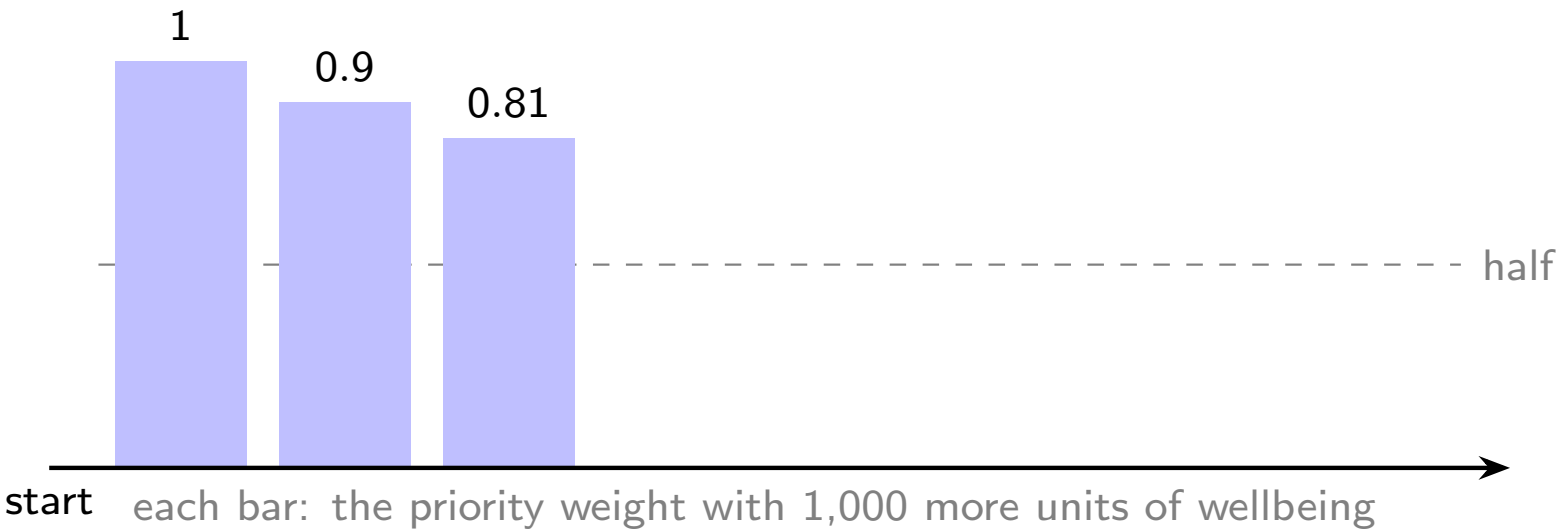
Start at any level: call the priority weight there 1.

Seven Steps Halve the Priority Weight



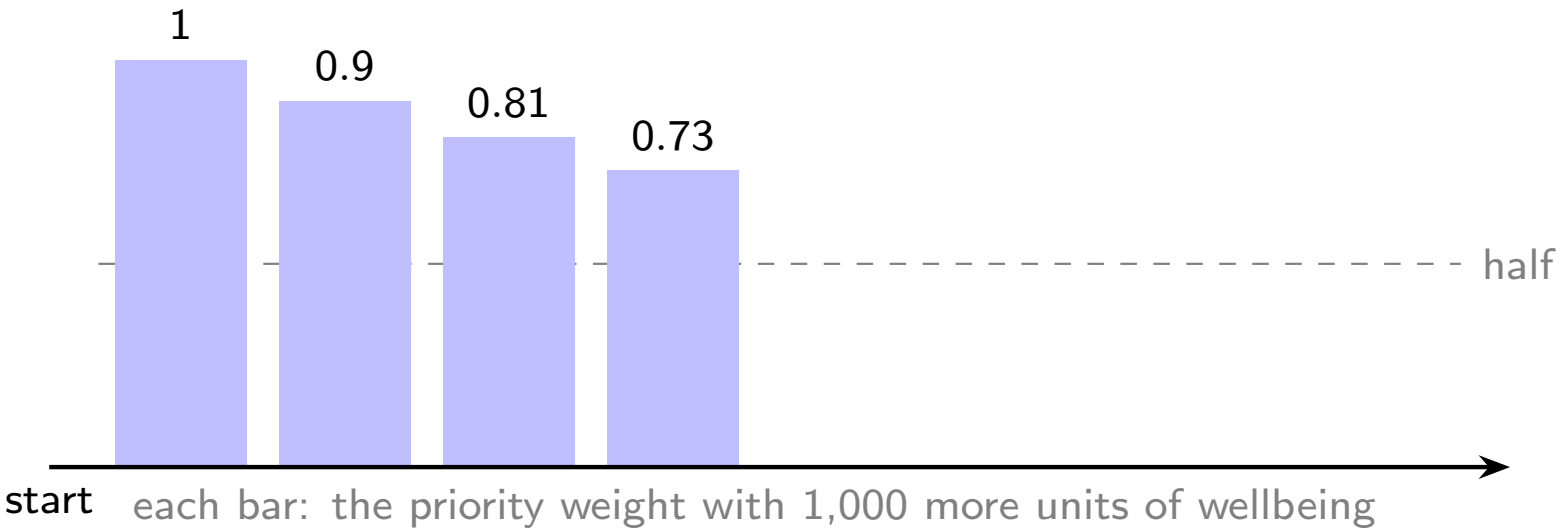
With 1,000 more units: $\times 0.9$.

Seven Steps Halve the Priority Weight



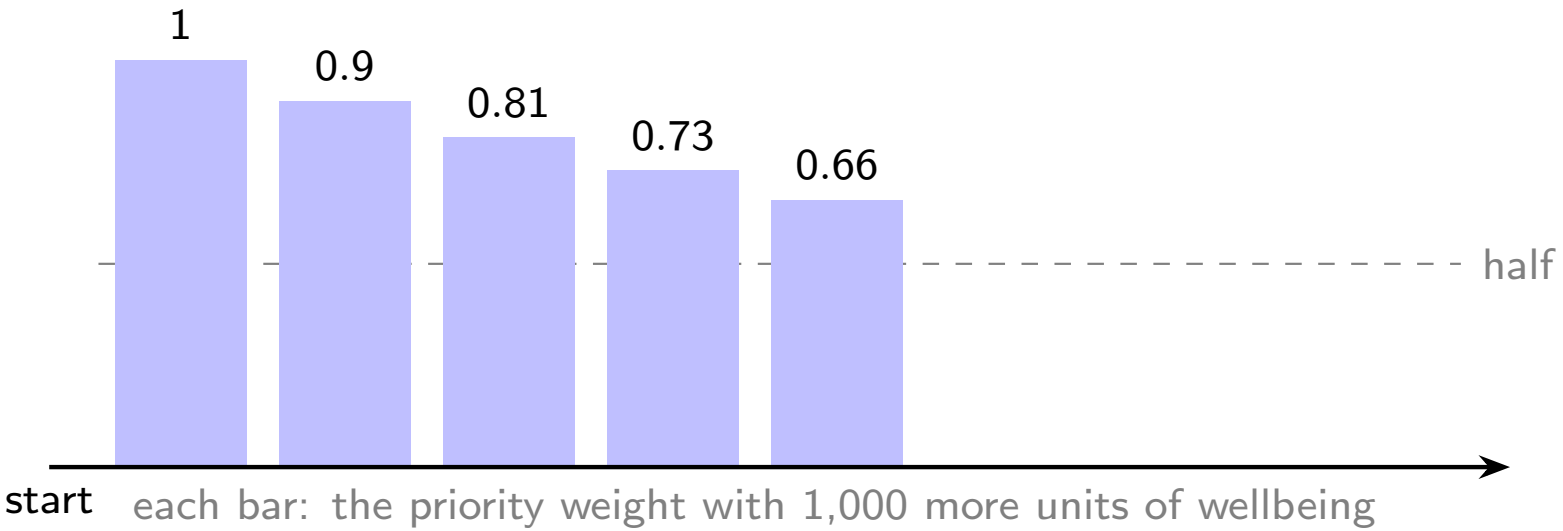
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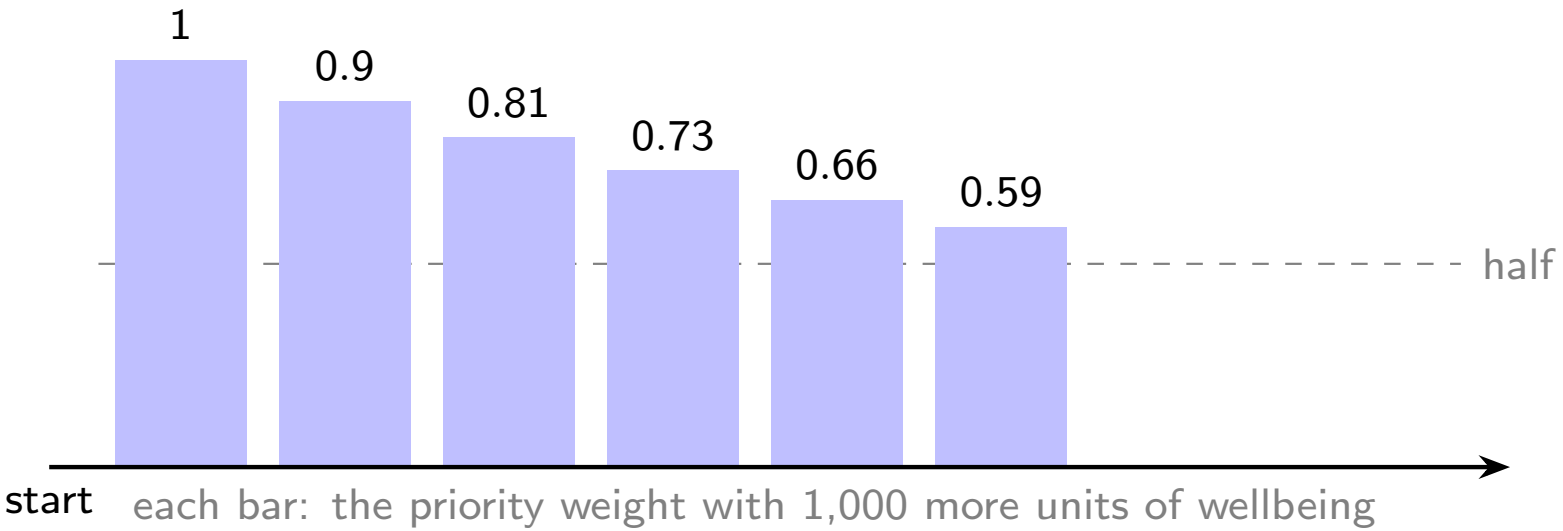
And again, with every 1,000 more units.

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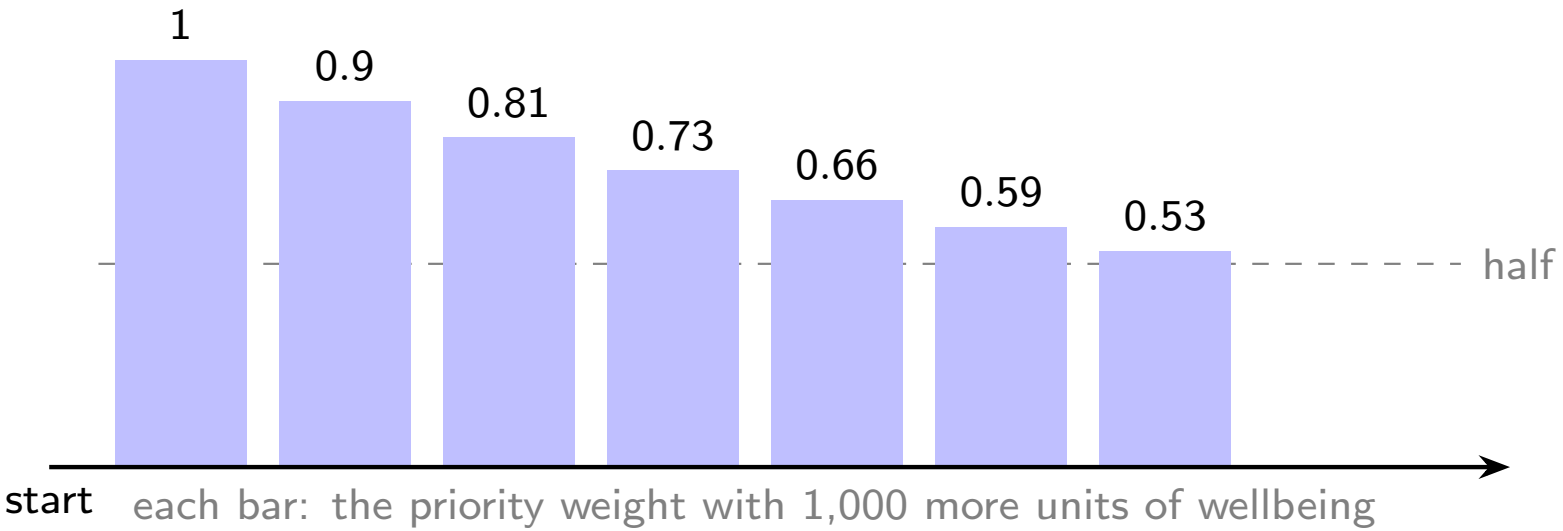
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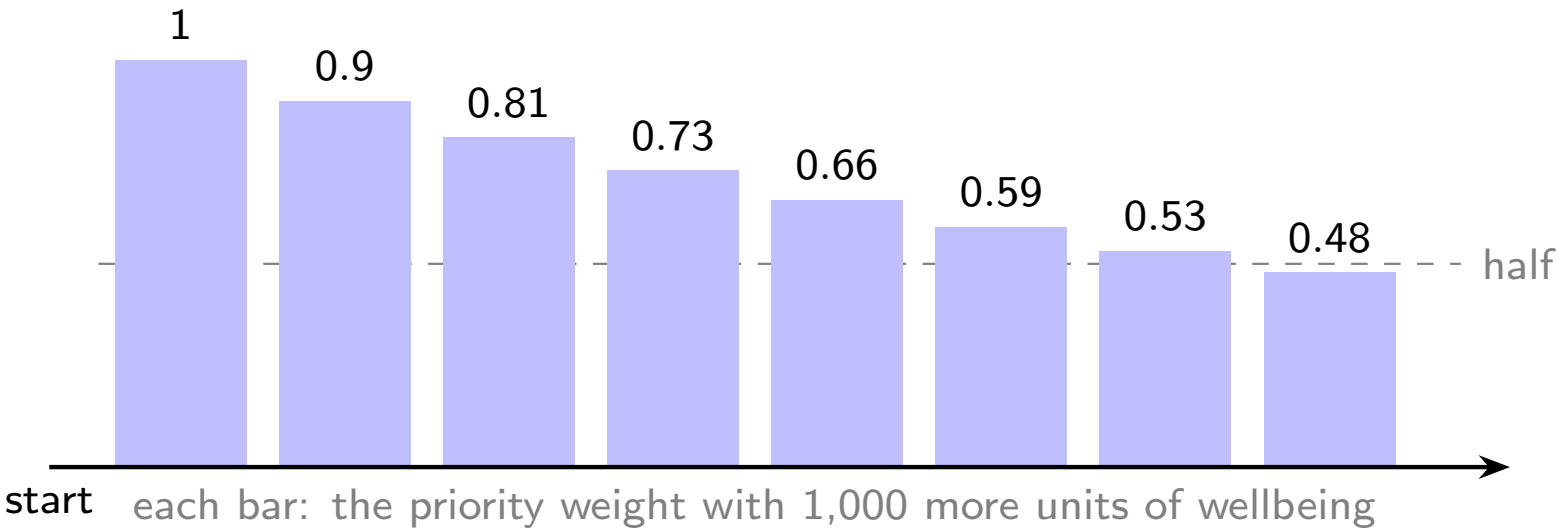
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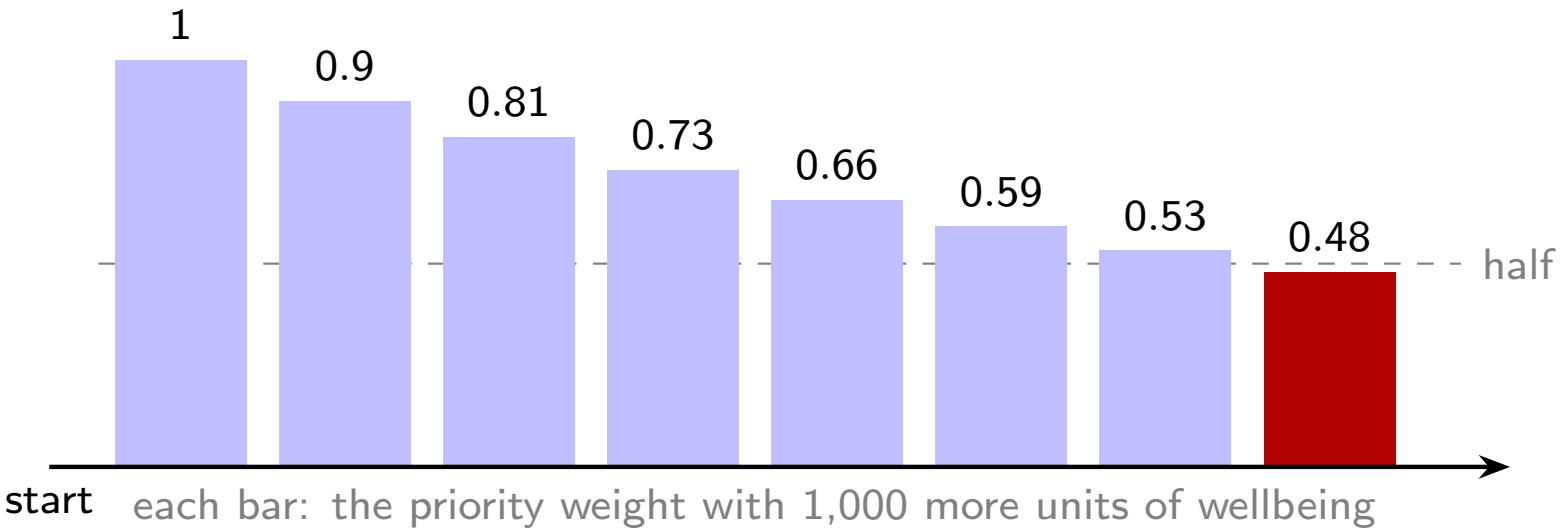
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After seven steps, with 7,000 more units: $0.9^7 \approx 0.48$, less than half.
So with every 7,000 more units, the priority weight at least halves.

Halving for Prioritarians



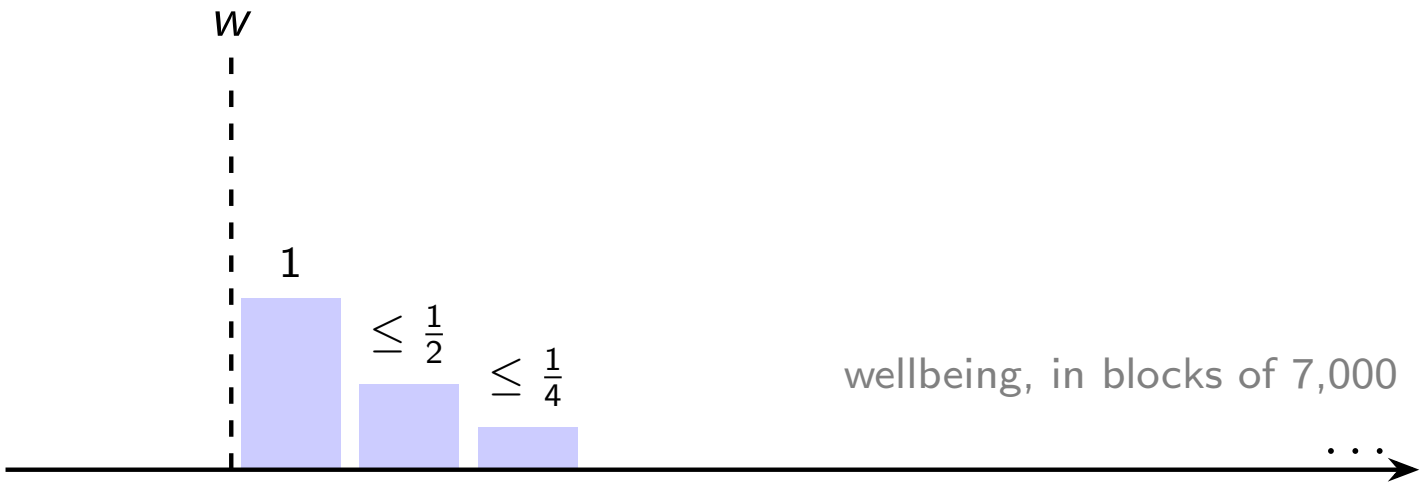
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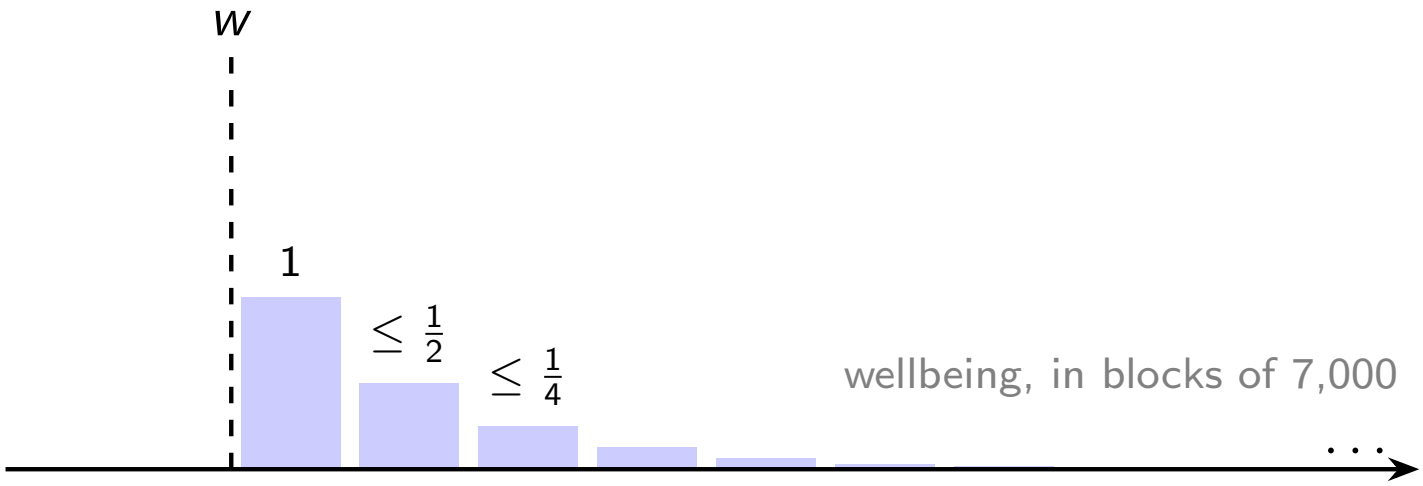
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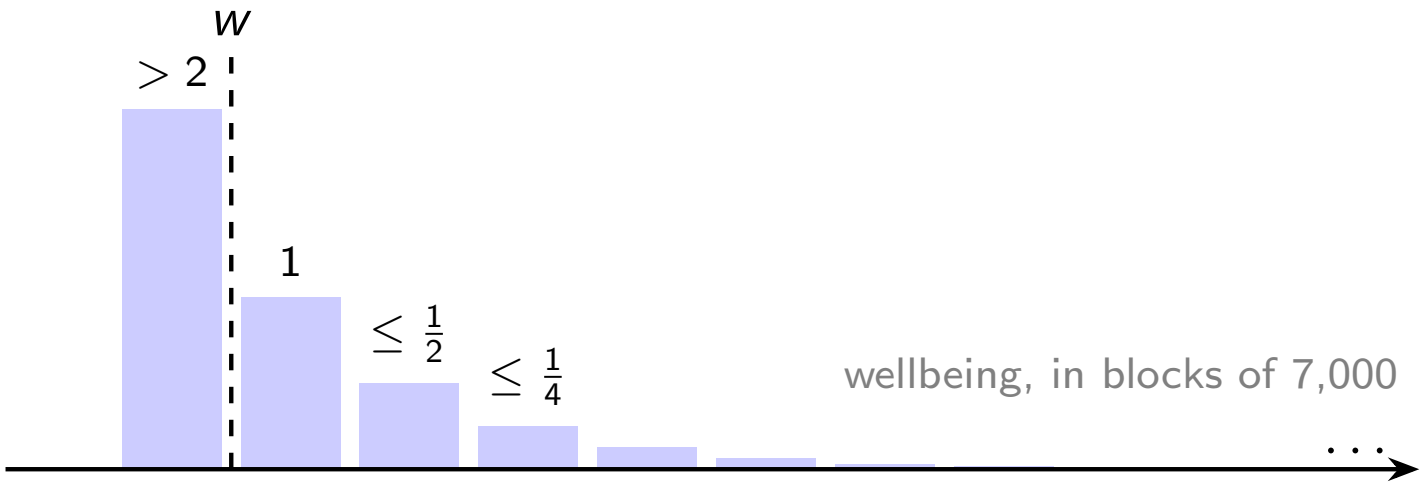
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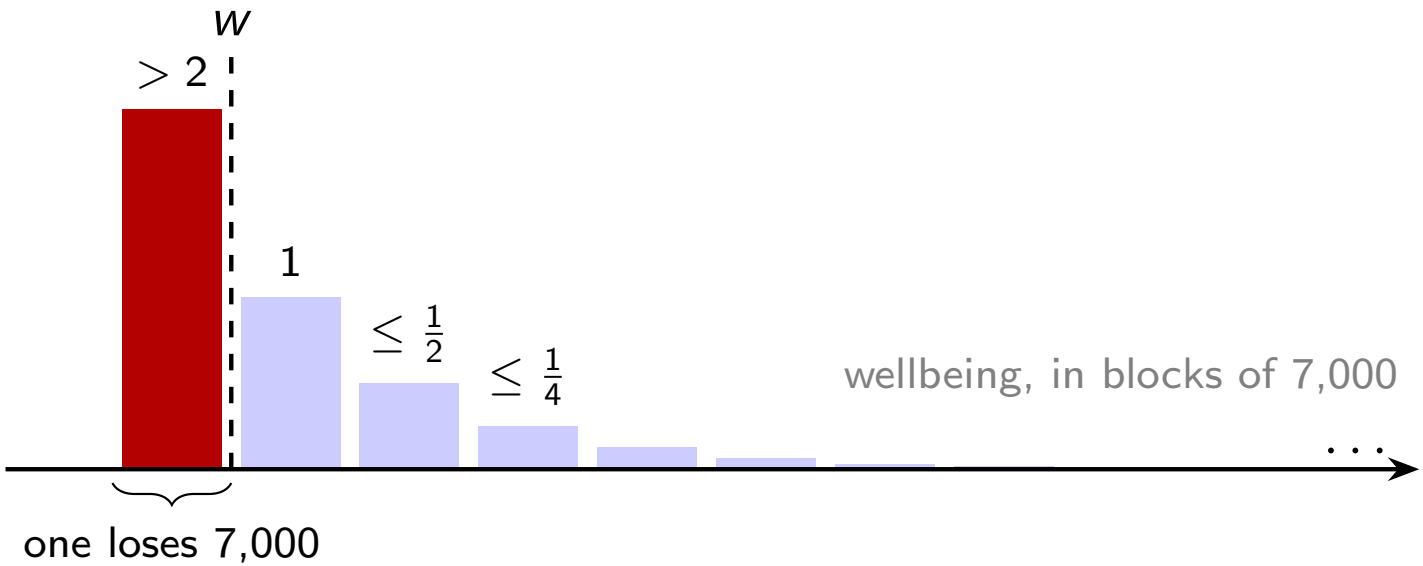
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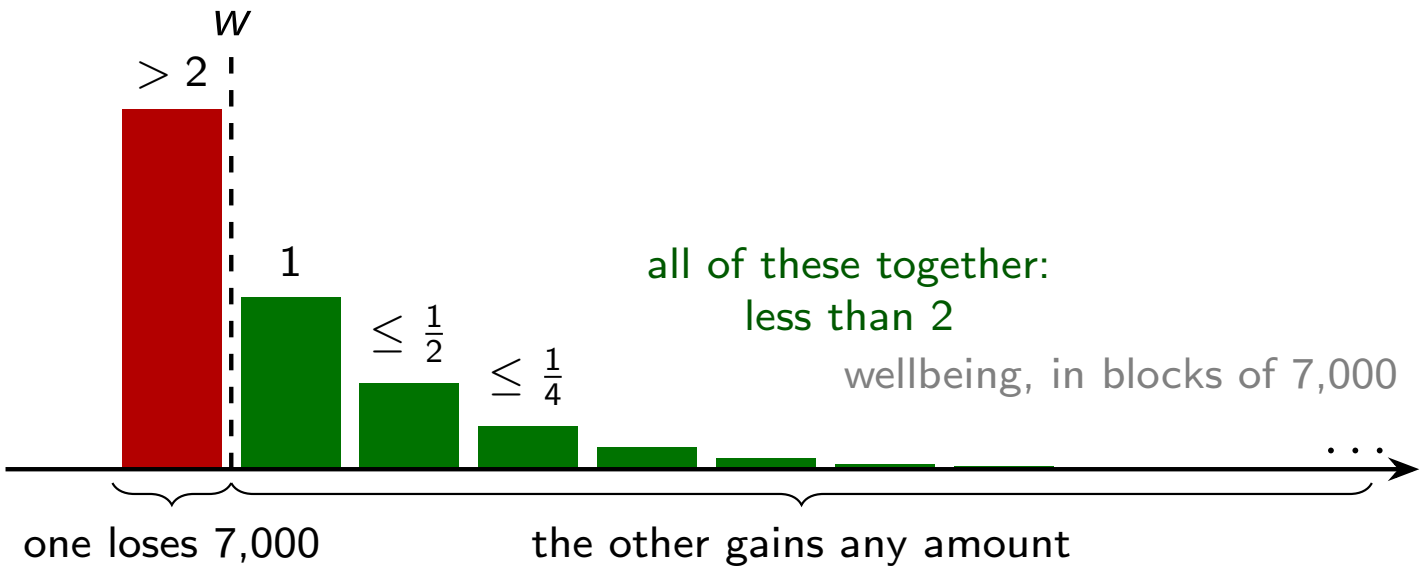
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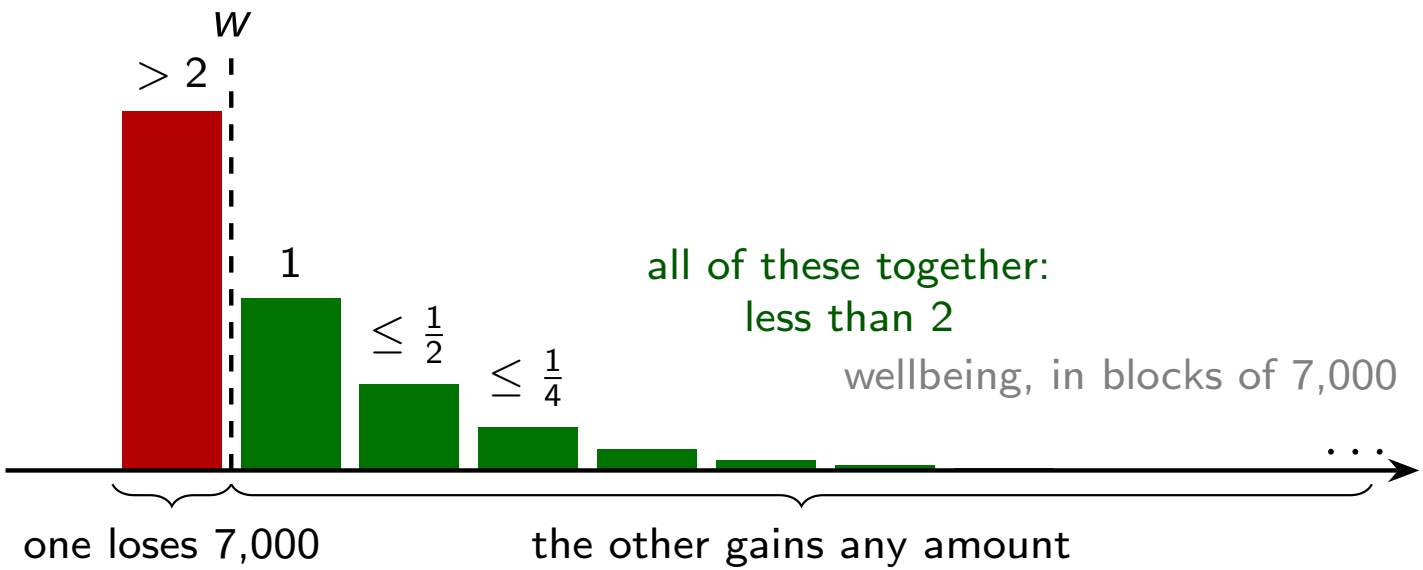
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So (w, w) beats $(w - 7{,}000, w + G)$, however large G is.

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- A prioritarian who prefers (w, w) to $(w - 1{,}000, w + 1{,}100)$ at every level must prefer (w, w) to $(w - 7{,}000, w + G)$, however large G is.

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- With $2n$ people, pair them off: $2n$ people at w beat n at $w - 7{,}000$ and n at $w + G$.
- As with risk, this works for any concave priority weighting. Nebel and Stefánsson (2023) make this point about prioritarianism.

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- The argument with numbers added up a priority-weighted number for each person. So it works against prioritarianism.

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- But an egalitarian needn't rank outcomes by adding up a number for each person.
- Adding up like that is separable. An egalitarian who denies Separability over Persons can't add up like that.
- So let's look at one egalitarian view that doesn't: Gini egalitarianism.

The Gini Coefficient

The Gini coefficient

$$\frac{S}{2n^2 w_A}$$

where S is the sum of the gaps $|w_i - w_j|$ over all i and j, w_i is person i's wellbeing, n is the number of people and w_A is the average wellbeing.

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- Then divide by twice the average wellbeing, $2w_A$, so that the scale doesn't matter.

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- Then divide by twice the average wellbeing, $2w_A$, so that the scale doesn't matter.
- It's 0 when everyone is equal, and close to 1 when one person has nearly everything.

The Gini Coefficient at Work

Alex at 2, Darius at 10, Philip at 20. Total 32, average $w_A = \frac{32}{3}$.

Gap	Alex	Darius	Philip
Alex			
Darius			
Philip			

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Alex	0	8	18
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Philip	18	10	0

The gap between each pair, counted both ways.

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$S = 72$. Divided by $n^2 = 9$: the average gap is 8.

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Divided by twice the average wellbeing, $2w_A = 2 \times \frac{32}{3} \approx 21.3$:
Gini coefficient $8 \div 21.3 = 0.375$.

Gini Egalitarianism

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Rank outcomes by total wellbeing $\times (1 - \text{Gini coefficient})$. (For positive wellbeing levels.)

Gini Egalitarianism

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Rank outcomes by total wellbeing $\times$ (1 $-$ Gini coefficient). (For positive wellbeing levels.)

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- Utilitarianism: equally good. Gini egalitarianism: EQUAL is better.
- Gini egalitarianism satisfies Pigou–Dalton and Continuity, but not Separability over Persons.

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- Three people: $\frac{1}{3}$, $\frac{3}{3}$, $\frac{5}{3}$. Four people: $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$.

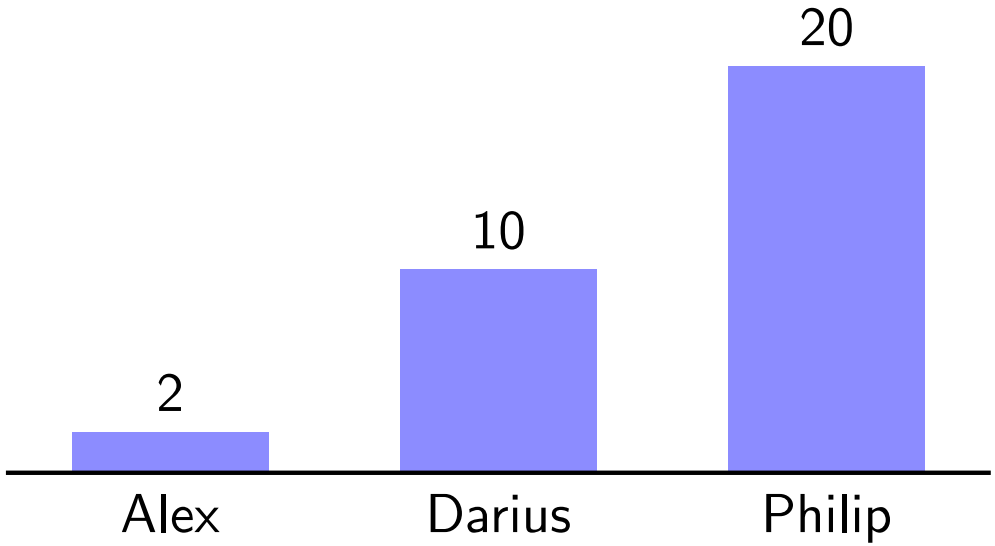
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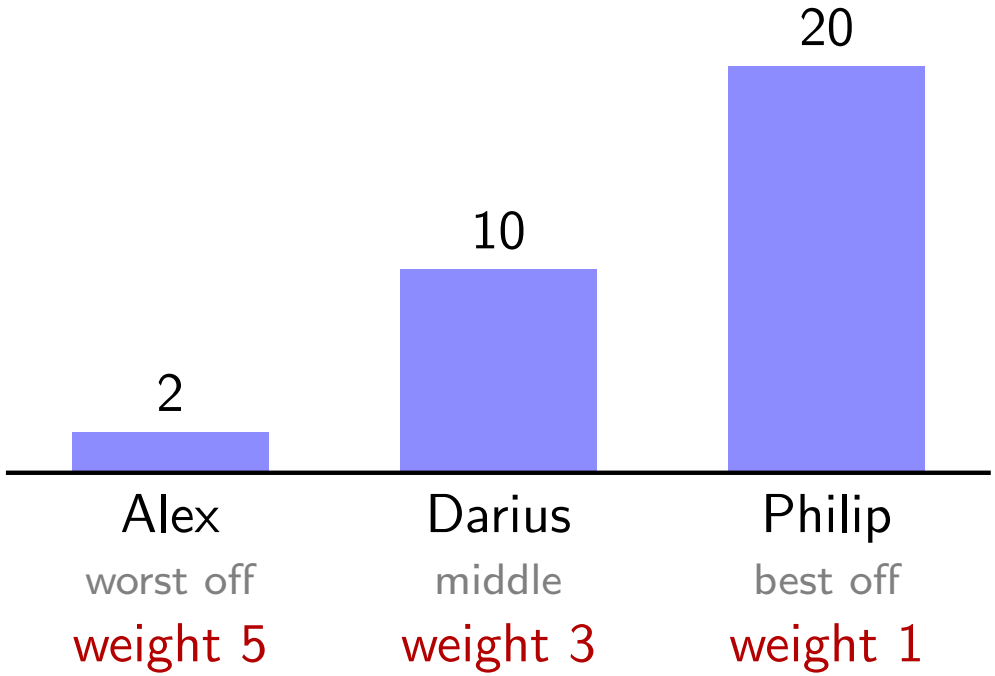
- Three people: $\frac{1}{3}$, $\frac{3}{3}$, $\frac{5}{3}$. Four people: $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$.
- The weight goes with your *rank*, not with you: the fewer people worse off than you, the bigger your weight.

Weighting by Rank



Alex at 2, Darius at 10, Philip at 20.

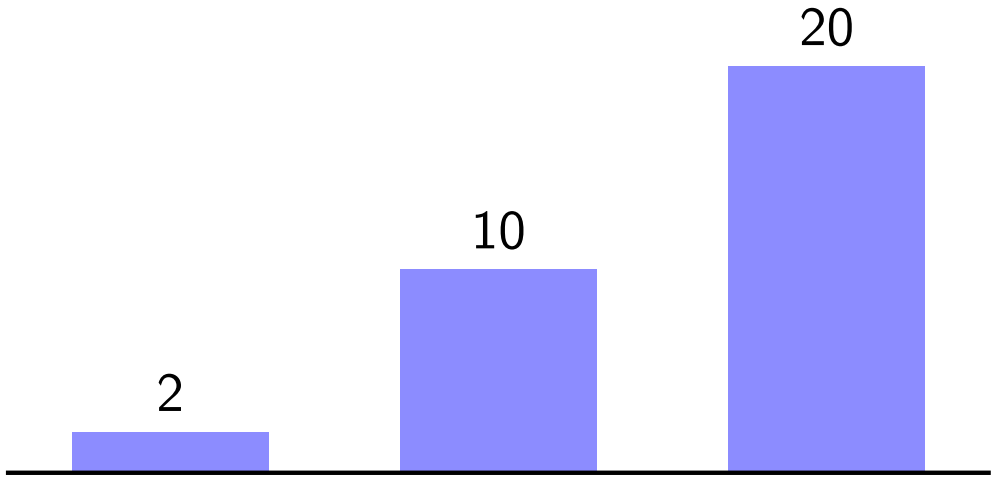
Weighting by Rank



Line them up from worst off to best off.

Weights $\frac{5}{3}$, $\frac{3}{3}$, $\frac{1}{3}$: to keep it simple, use 5, 3, 1 and divide by 3 at the end.

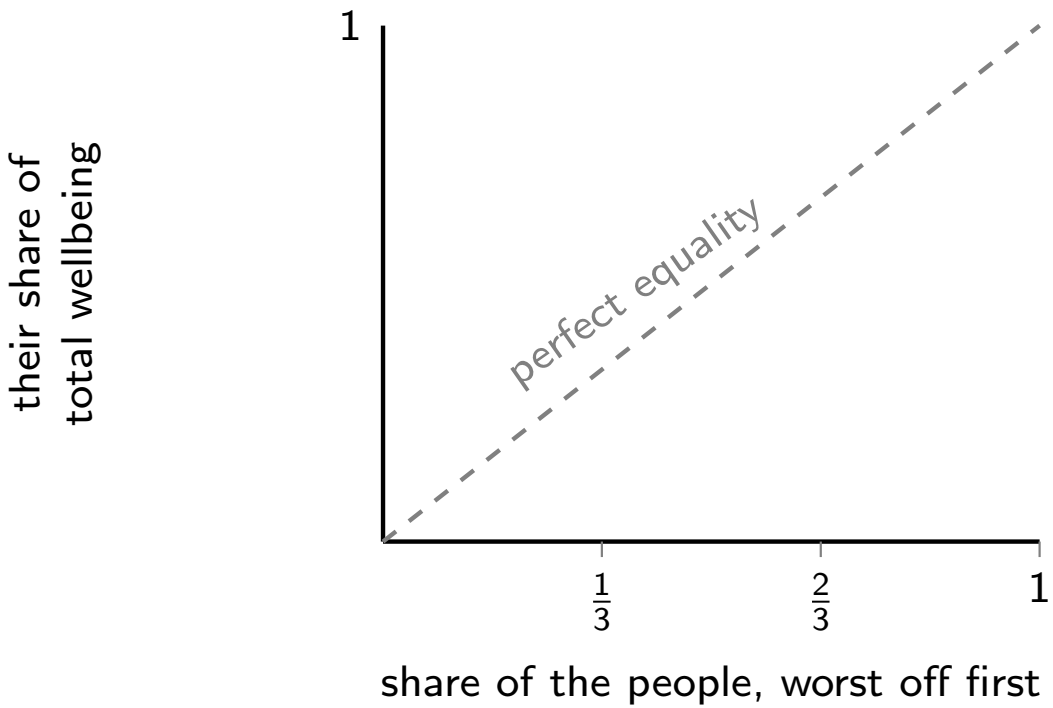
Weighting by Rank



total 60
$\div 3 = 20$

The weighted sum is $60 \div 3 = 20$.
The same as total $\times$ (1 $-$ Gini coefficient): $32 \times (1 - 0.375) = 20$.

Picturing the Gini Coefficient



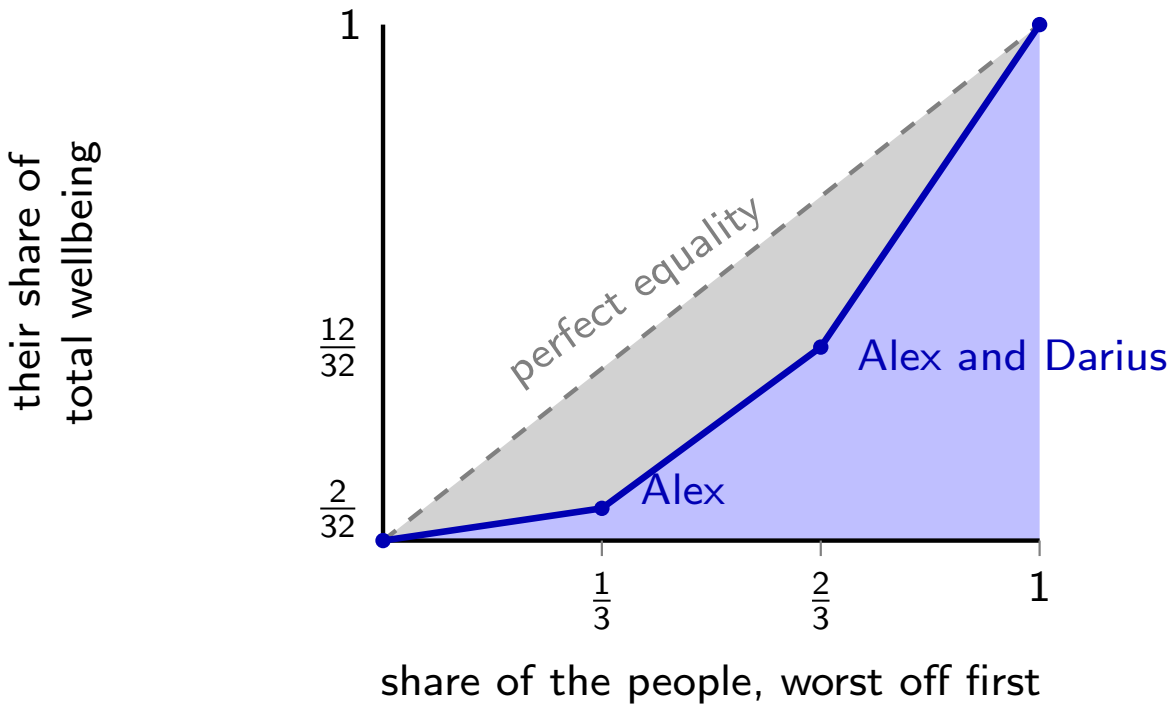
Line people up, worst off first. Across: share of the people. Up: their share of total wellbeing. Perfect equality is the diagonal.

Picturing the Gini Coefficient



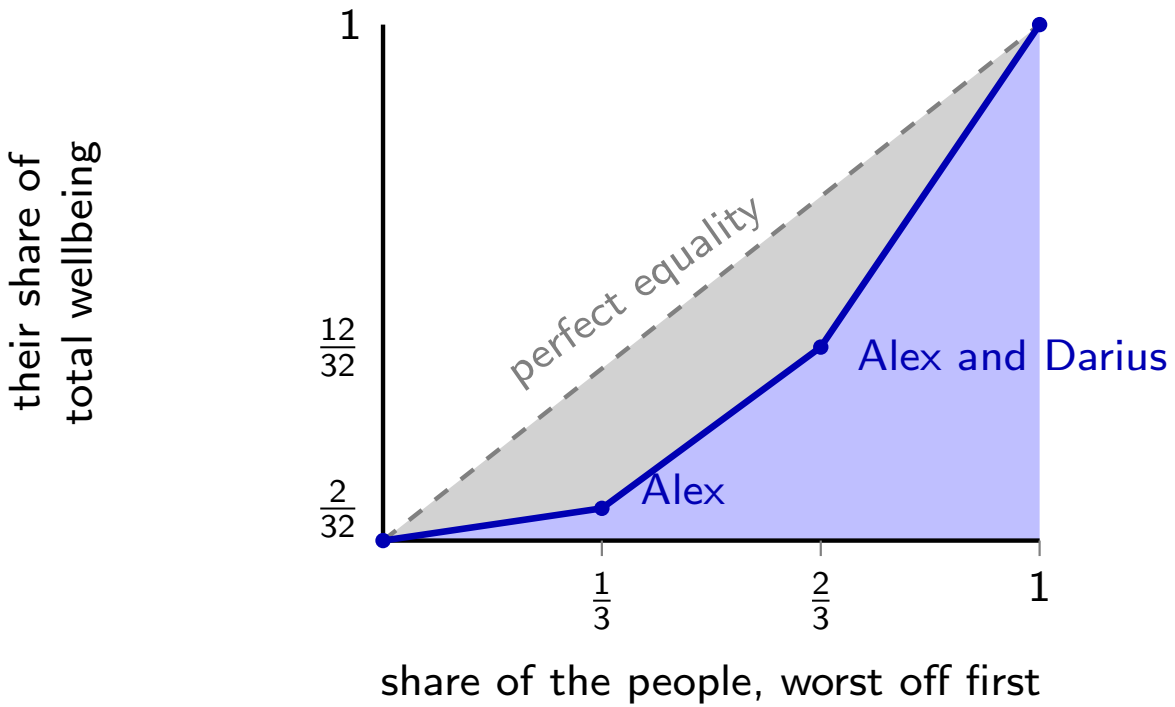
Alex at 2, Darius at 10, Philip at 20, out of 32: Alex has $\frac{2}{32}$; Alex and Darius together have $\frac{12}{32}$. That's the Lorenz curve.

Picturing the Gini Coefficient



Gini coefficient: the grey gap as a fraction of the whole triangle under the diagonal: $0.1875 \div 0.5 = 0.375$.

Picturing the Gini Coefficient



The grey gap is the share of total wellbeing "lost to inequality": $32 \times (1 - 0.375) = 20$, so 12 of the 32 are lost. The blue part still counts.

Gini and Separability

Score: weights 5, 3, 1 from worst off to best off (3 $\times$ the Gini value, so the same ranking).

	Alex	Darius	Philip	Score
X	6	6	1	$5 \cdot 1 + 3 \cdot 6 + 1 \cdot 6 = 29$
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- With Philip at 20, their weights are 5 and 3, a ratio of $\frac{5}{3}$: the gain only has to beat $\frac{5}{3} \times 4 \approx 6.7$. Y' is better.

Does Gini Egalitarianism Escape?

Two people: weights $\frac{3}{2}$ and $\frac{1}{2}$.
Value $= (3 \times$ worse off $+ 1 \times$ better off$) \div 2$.

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- But one losing 7,000 and the other gaining G beats equality as soon as G is more than 21,000.
- So Gini egalitarianism seems to escape the calibration problem.

Gini With Other People Around

Now add ten more people, each at 50,000. Twelve people: the two worst off get weights $\frac{23}{12}$ and $\frac{21}{12}$.

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- So the escape is not robust.

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Giving \$2,000 to the richer person adds more money in total, but leaves the two more unequal.

Small Inequalities Aversion (2)

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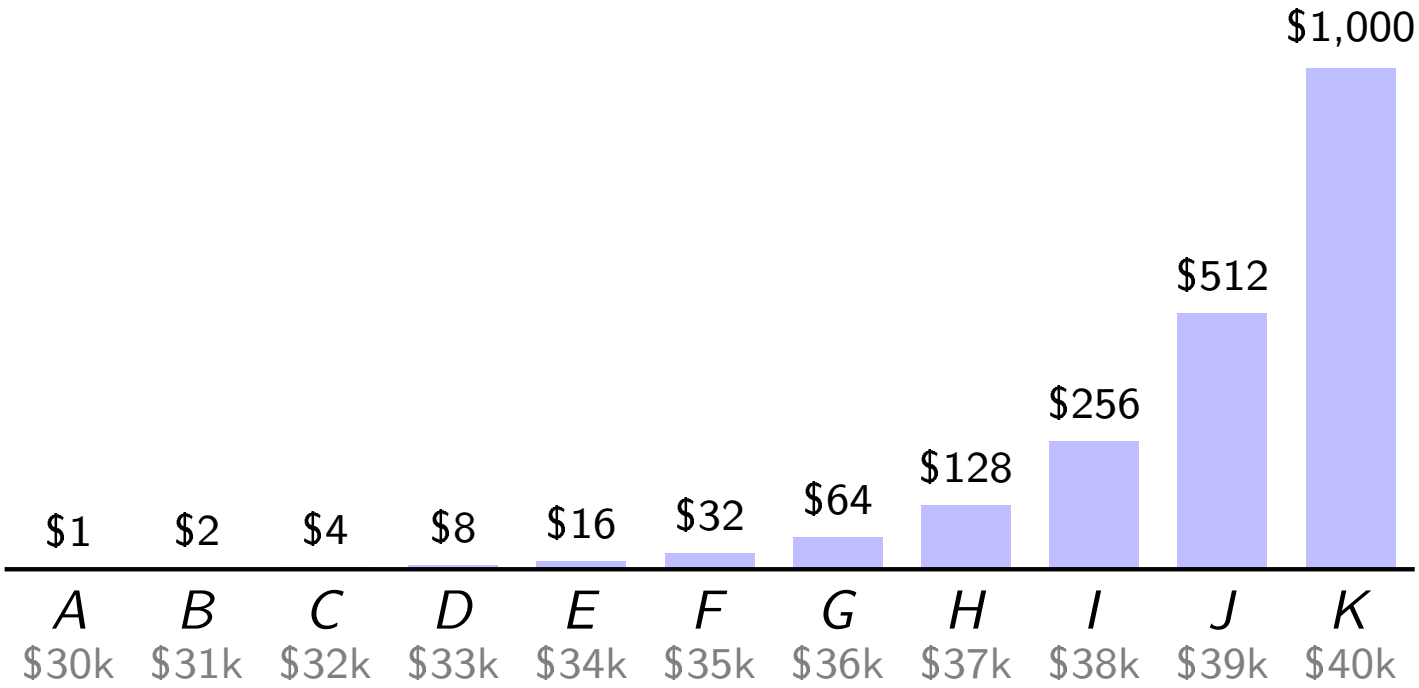
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- The background population: everyone else, who gets nothing either way.

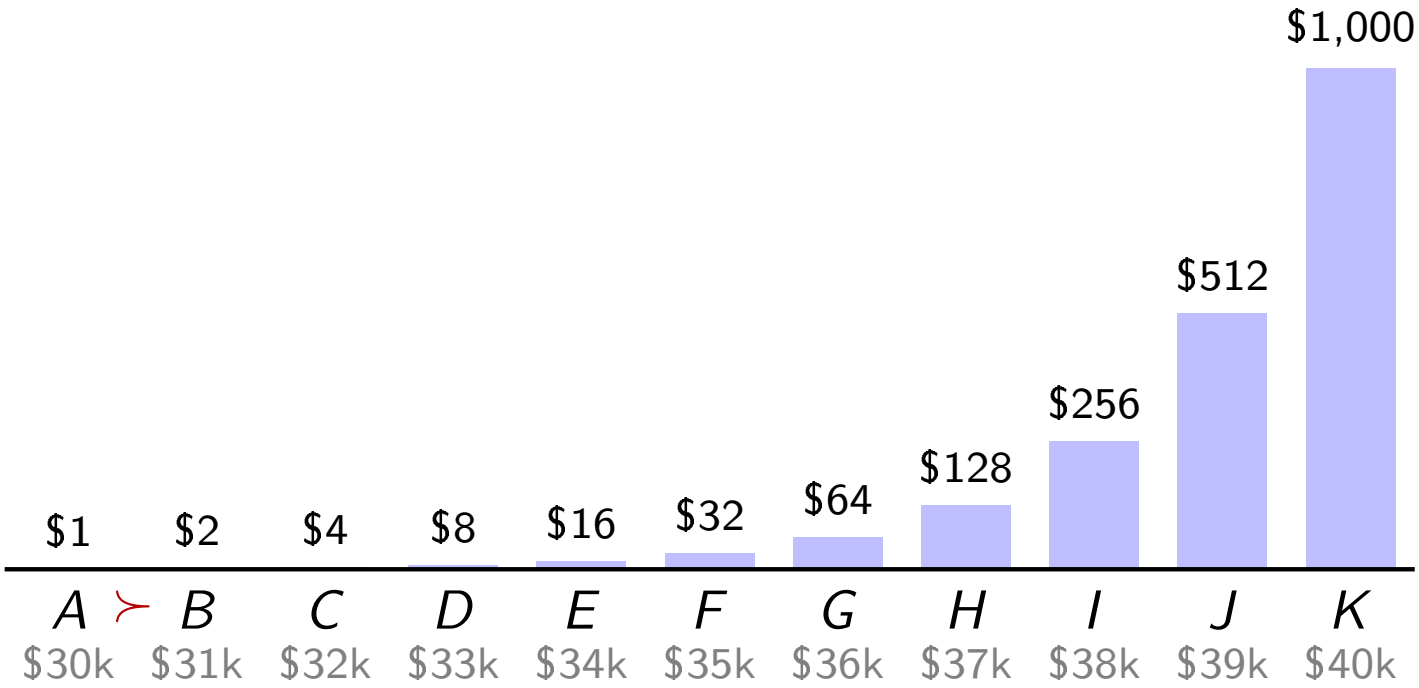
A Chain



Eleven people, A to K, \$1,000 apart in wealth. Each gift is double the last.

Each option gives *one* person their gift.

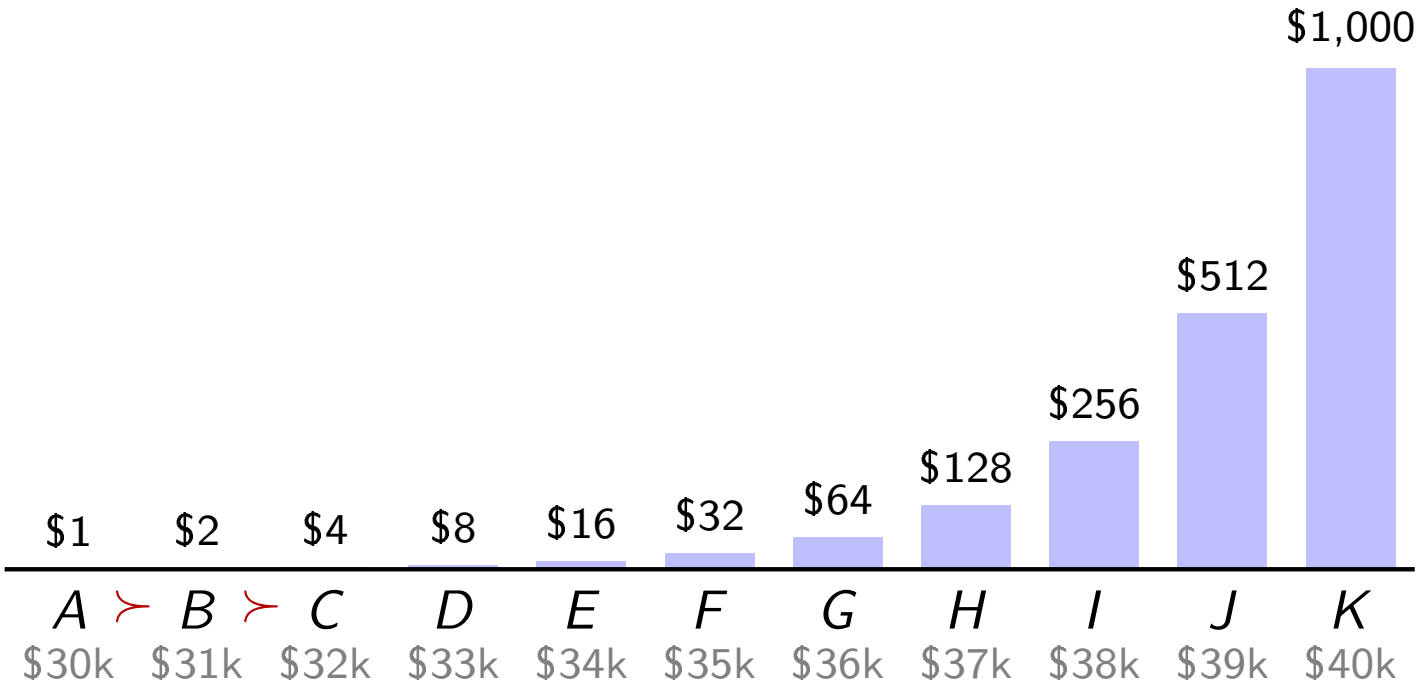
A Chain



B has \$1,000 more than A.

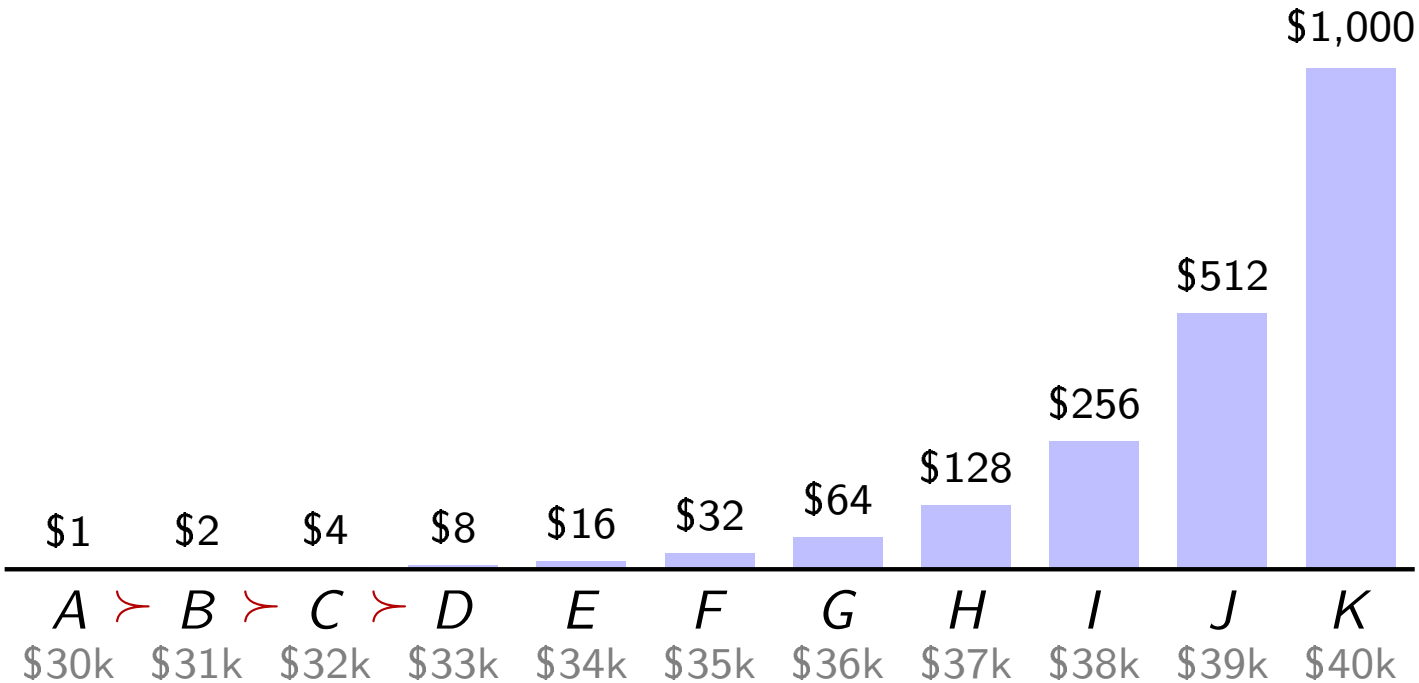
Small inequalities aversion: \$1 to $A \succ$ \$2 to B.

A Chain



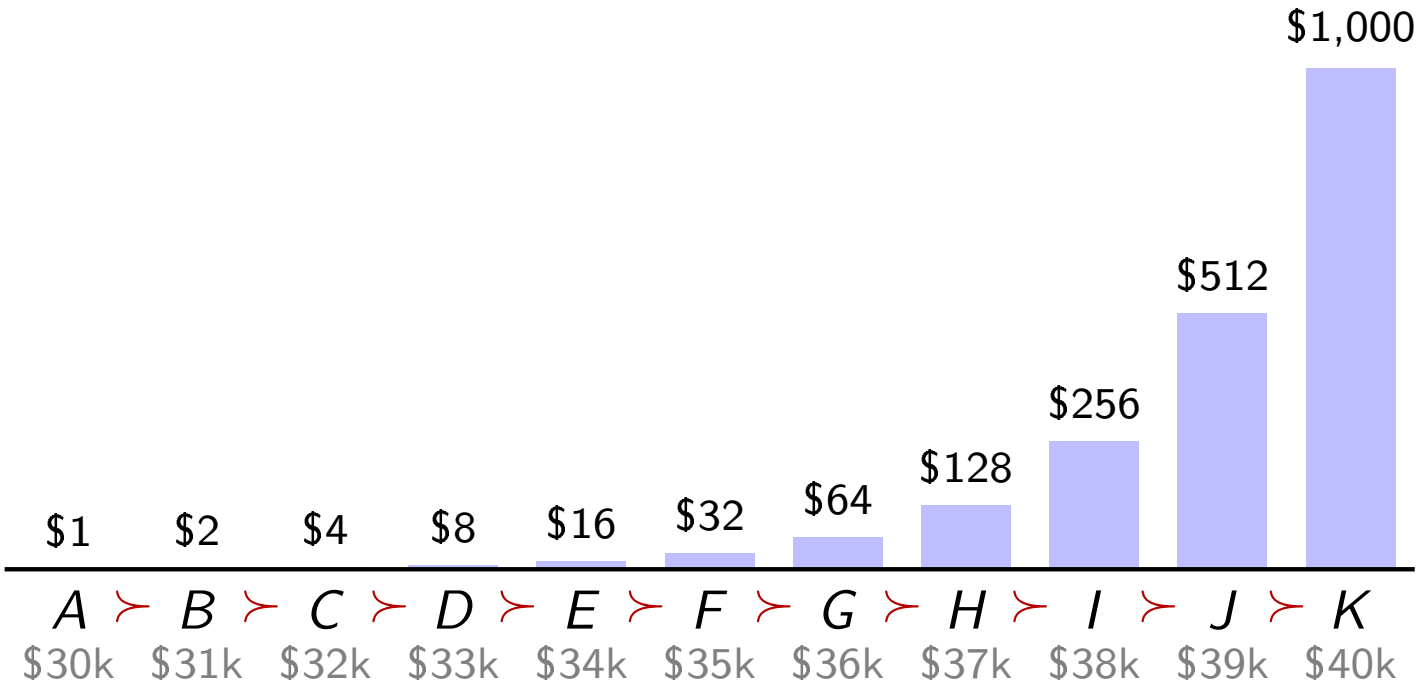
The same between B and C: \$2 to B $\succ$ \$4 to C.

A Chain



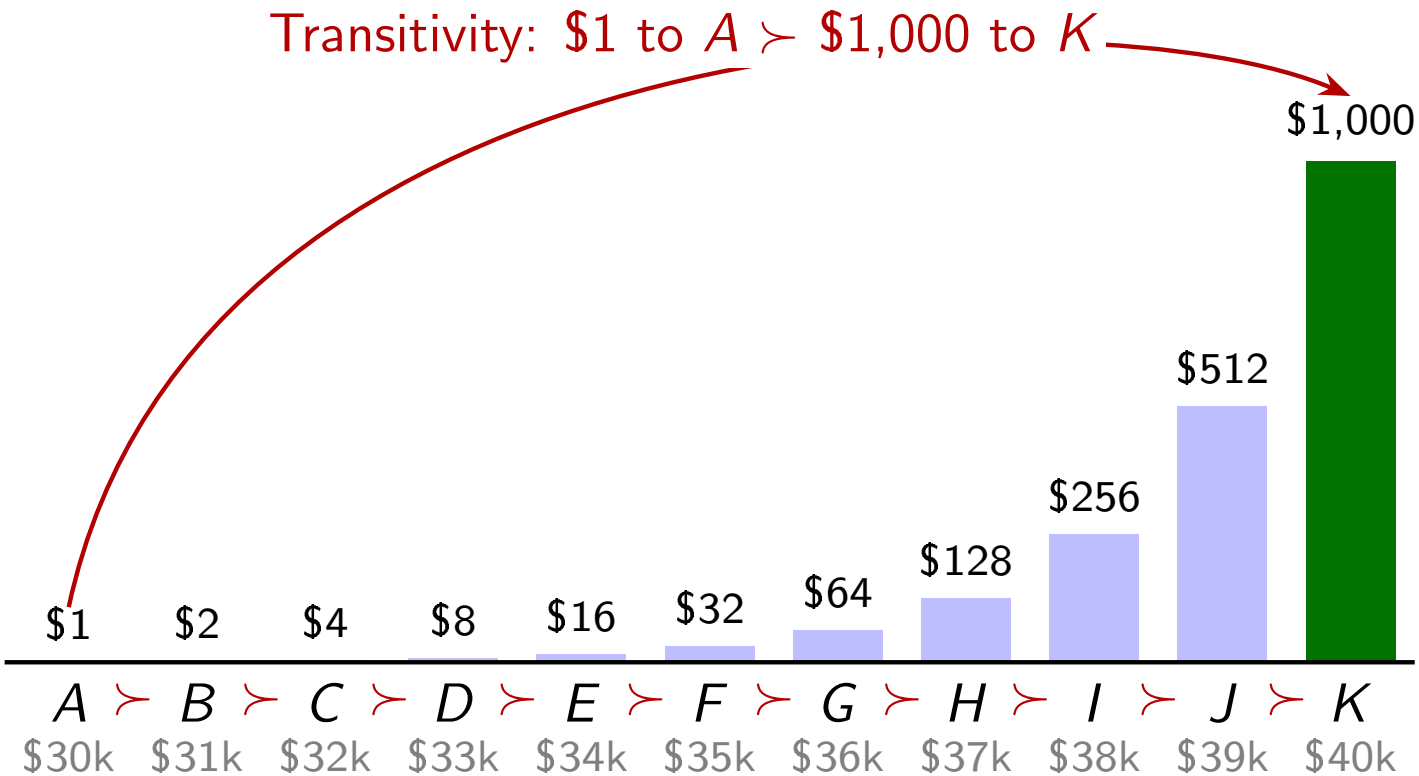
And between C and D: \$4 to $C \succ$ \$8 to D.

A Chain



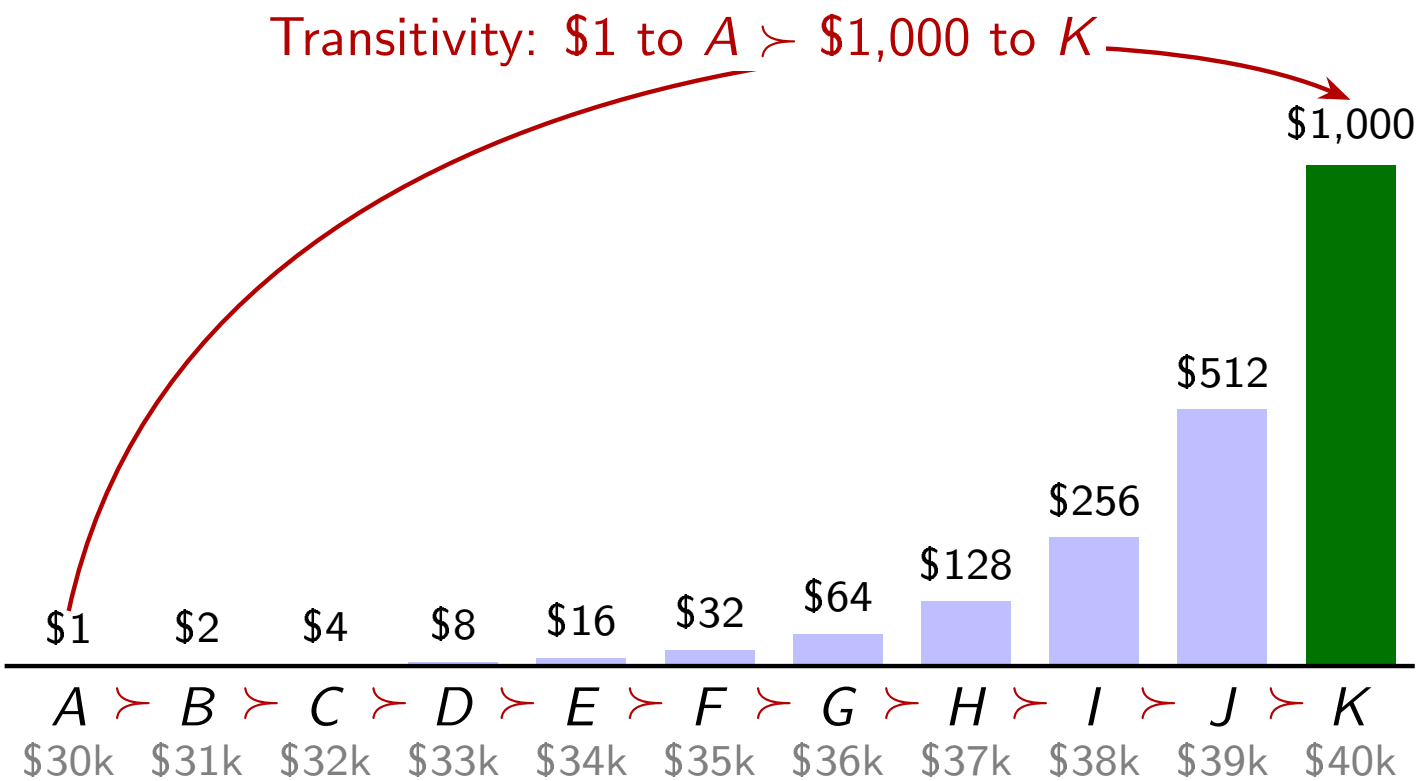
And so on up the line, to \$512 to $J \succ$ \$1,000 to K.

A Chain



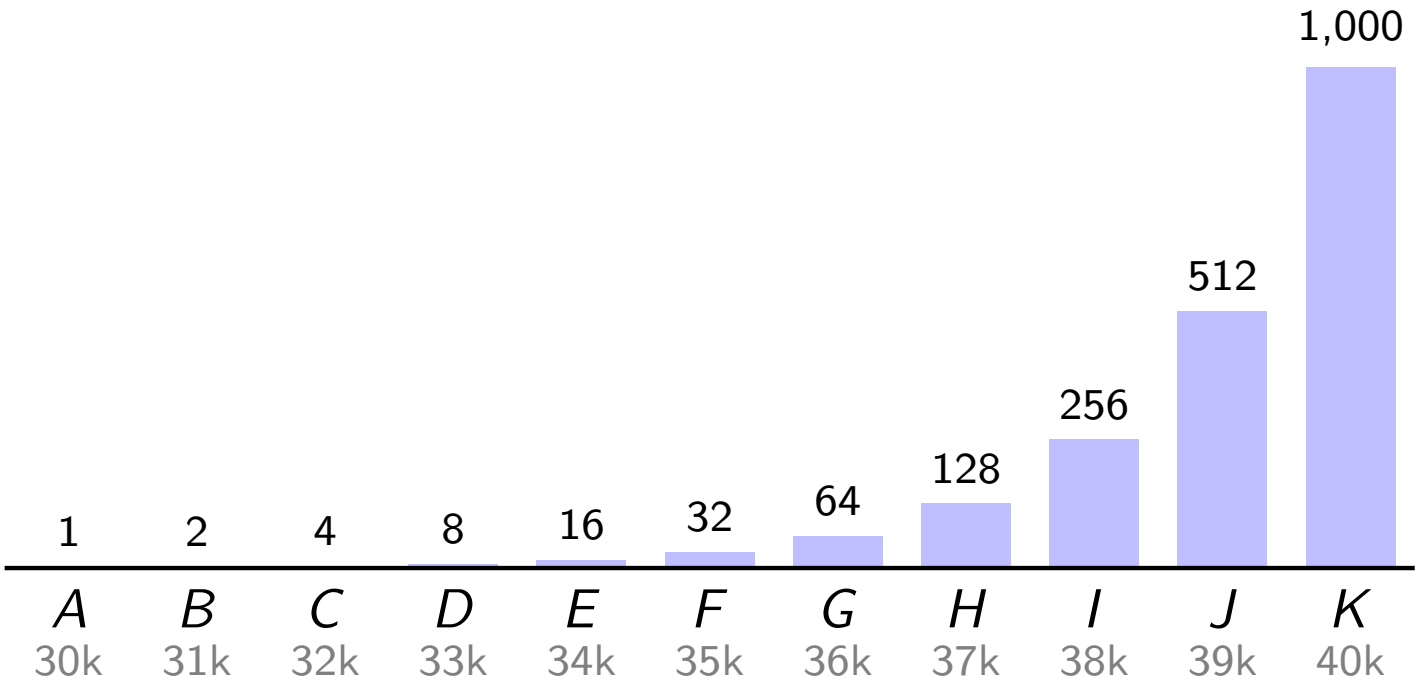
Transitivity: giving \$1 to A is better than giving \$1,000 to K.

A Chain



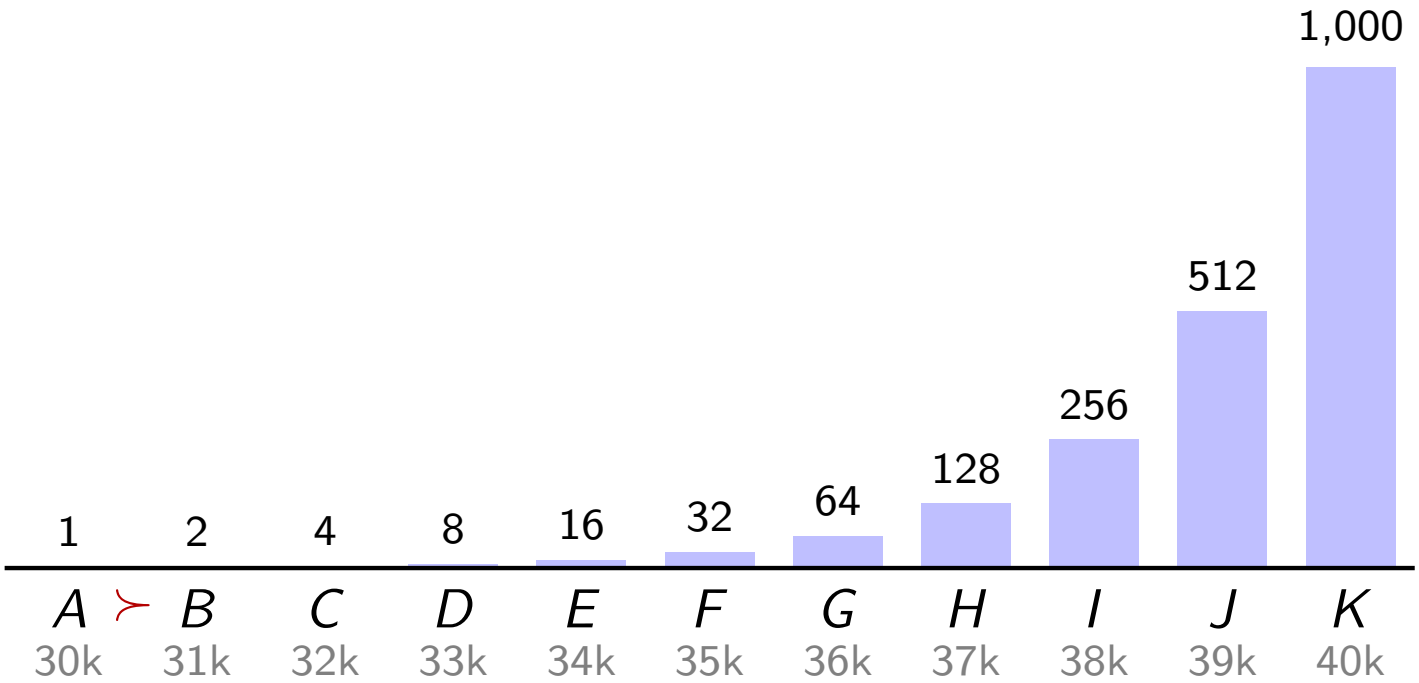
But it seems that the opposite is true.

The Same Chain in Wellbeing



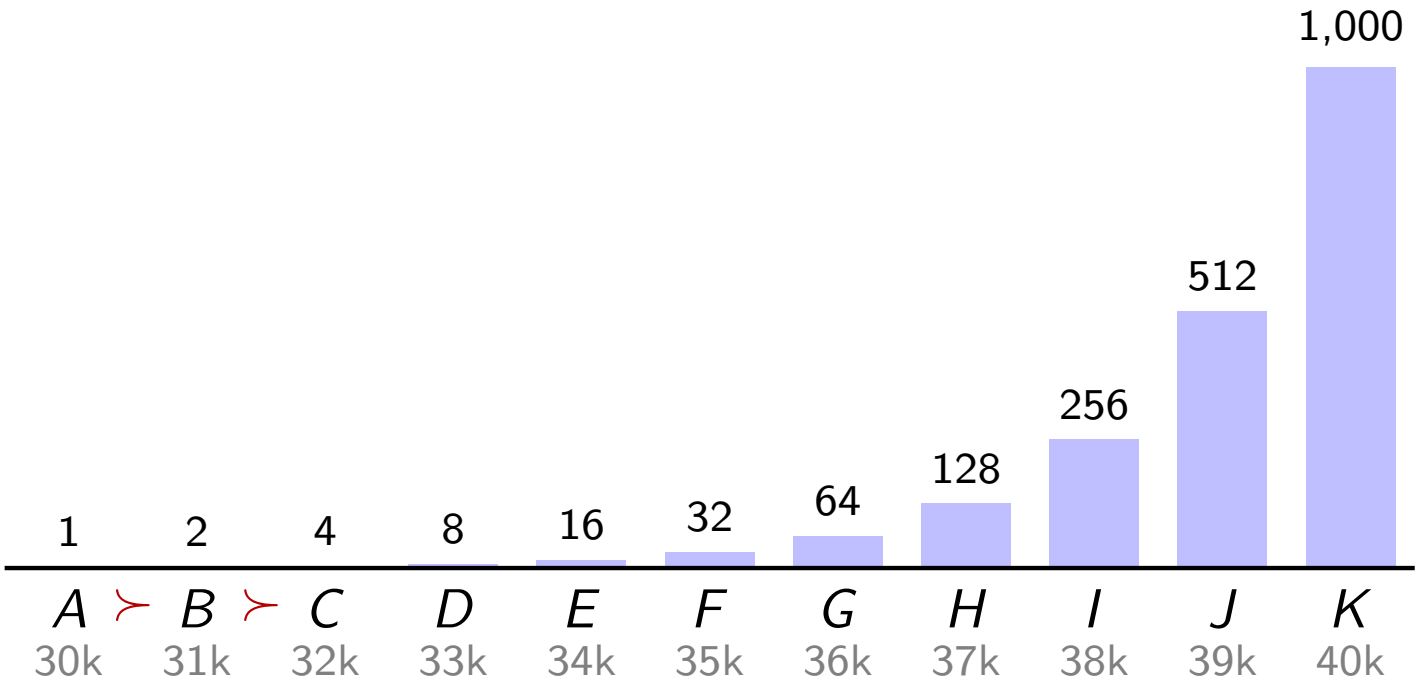
Now the amounts are units of wellbeing.
Eleven people, 1,000 units apart; each gift double the last.

The Same Chain in Wellbeing



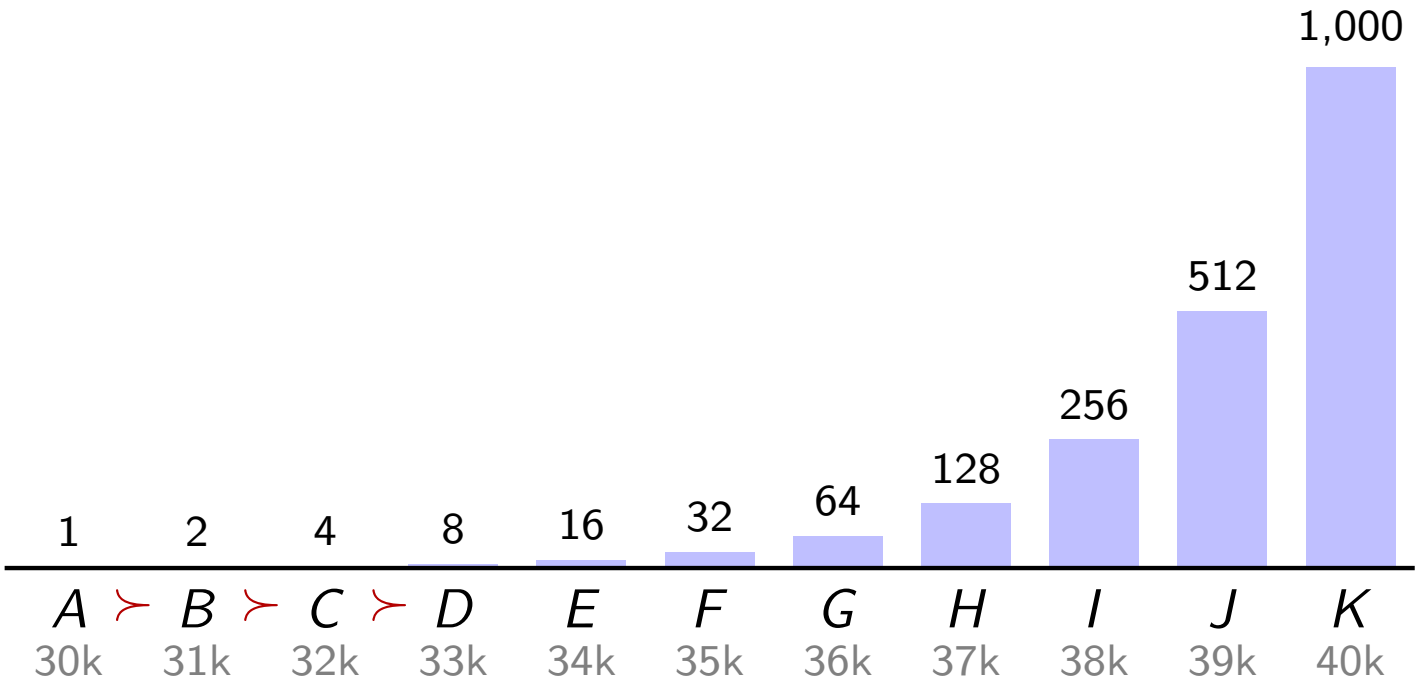
Small inequalities aversion, now about wellbeing: 1 to $A \succ$ 2 to B.

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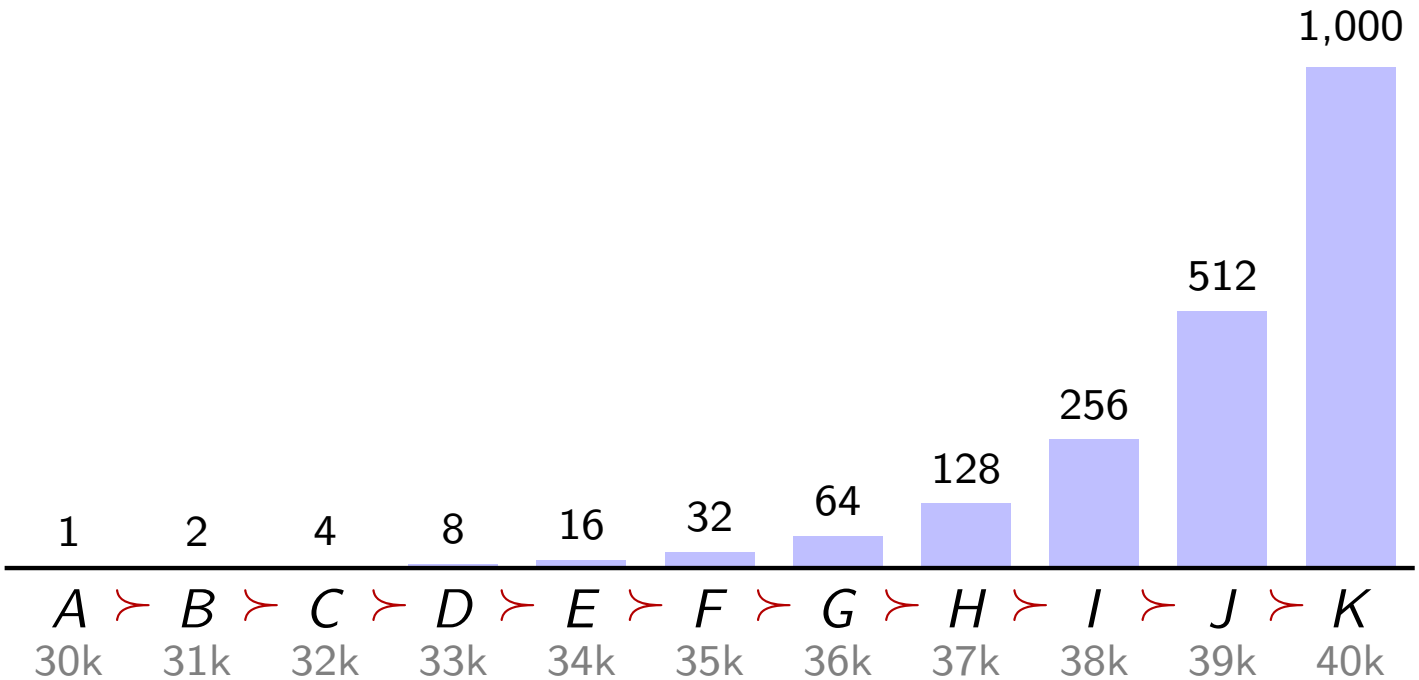
2 to $B \succ$ 4 to C.

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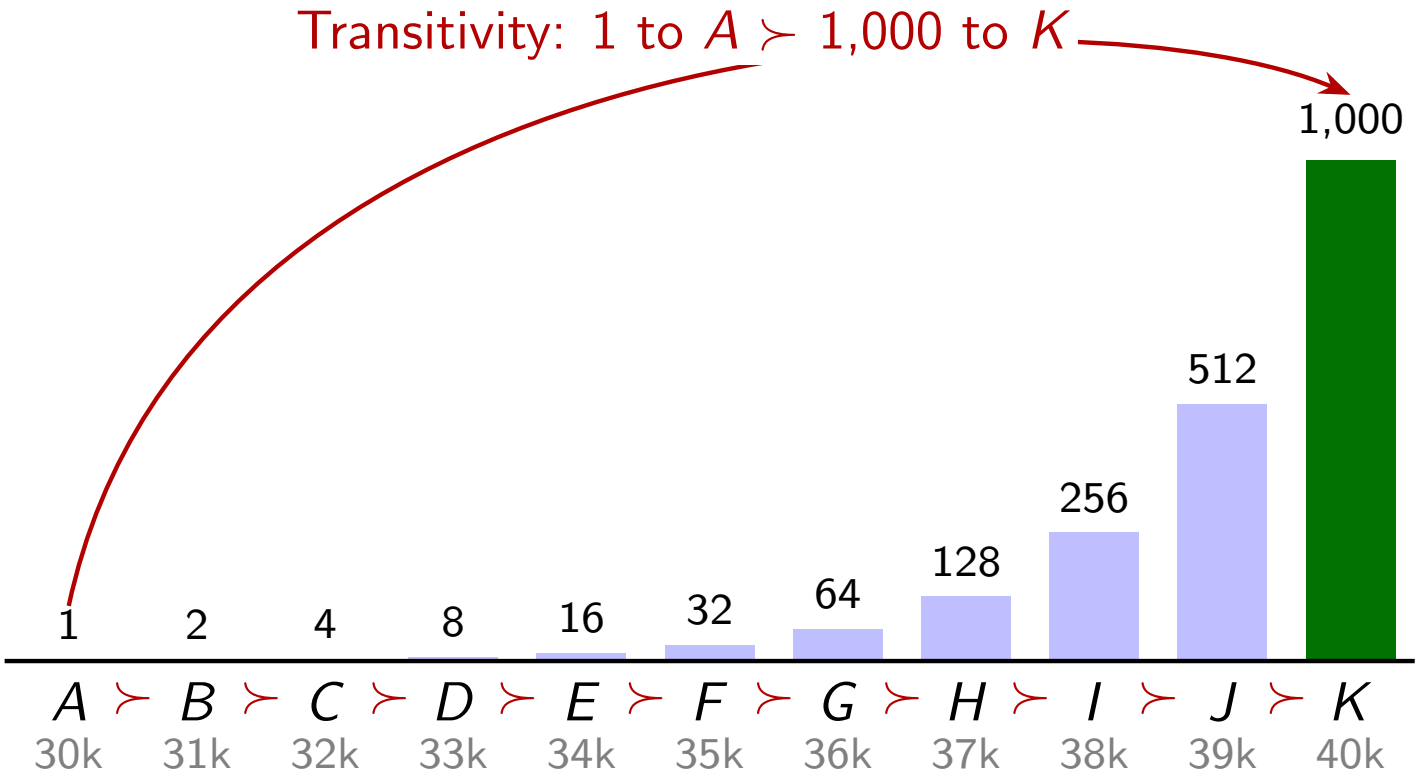
4 to $C \succ$ 8 to D.

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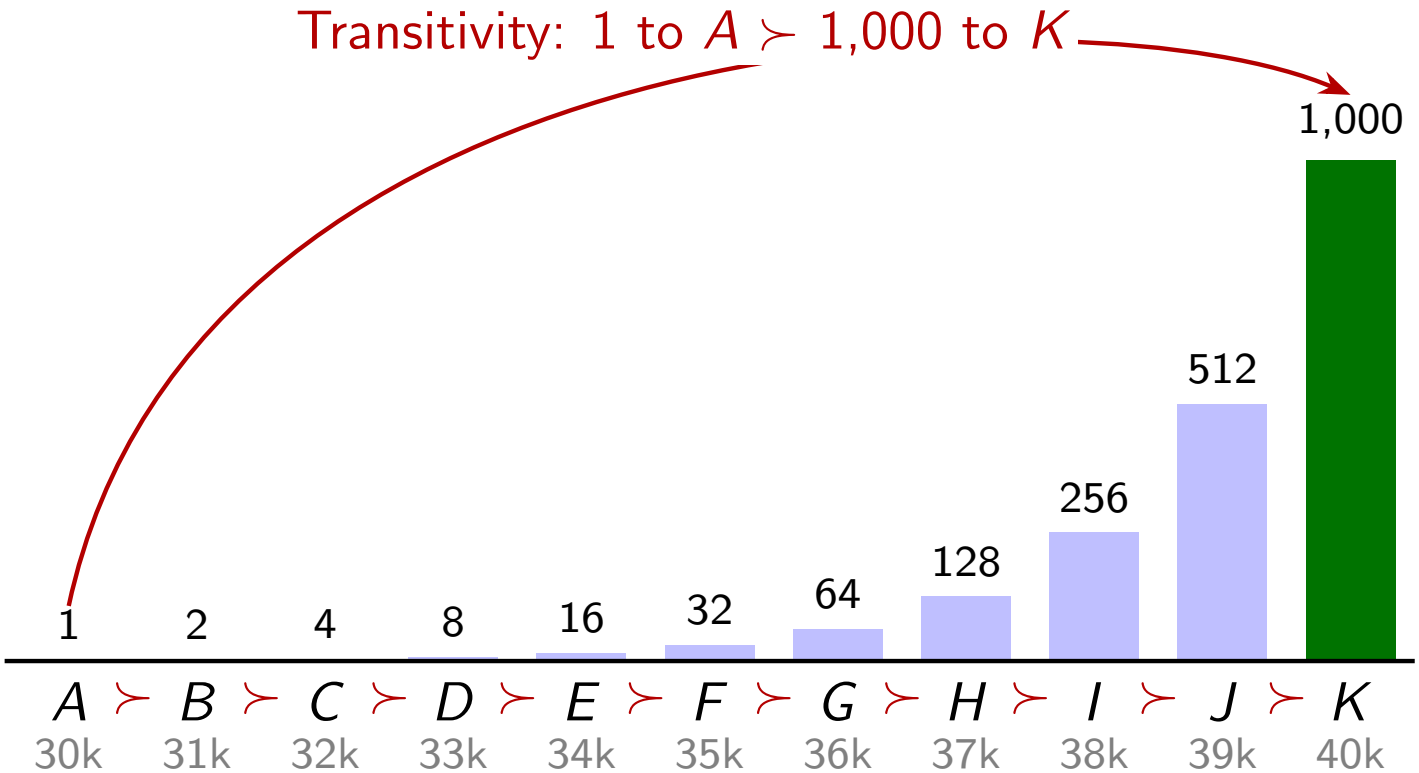
And so on, to 512 to $J \succ$ 1,000 to K.

The Same Chain in Wellbeing



Transitivity: 1 unit to A is better than 1,000 units to K.

The Same Chain in Wellbeing



So the same problem arises for prioritarians and egalitarians.

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- The chain uses only small inequalities aversion and Transitivity: no sum, no concave function.
- It doesn't assume any numbers at all: no utility function, no priority weighting. Only these inequality-aversion conditions.
- So the problem isn't just for prioritarians. It's a problem for all transitive rankings of distributions: egalitarian and utilitarian ones too.

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Non-extreme larger inequalities aversion	\$1,000 to $K \succ$ \$1 to A

Non-extreme larger inequalities aversion:
a big enough benefit to someone better off
can outweigh a small benefit to someone worse off.

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Claim	In the chain
Small inequalities aversion	\$1 to $A \succ$ \$2 to B, and so on
... regardless of background	whoever else is in the line
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Non-extreme larger inequalities aversion	\$1,000 to $K \succ$ \$1 to A

The chain shows that the four can't all hold.
Each way out gives up one of them.

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Rankings Relative to Option Sets

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- Losses and gains are measured from a reference level, such as where the person would be in the most equal feasible distribution.
- Reference levels depend on which options are available, so the ranking of two options can differ from one option set to another.
- Another: count only inequalities among those affected by the choice. Perhaps inequalities are only bad in themselves when they are avoidable.

What Rabin Concludes

Rabin (2000: 1282)

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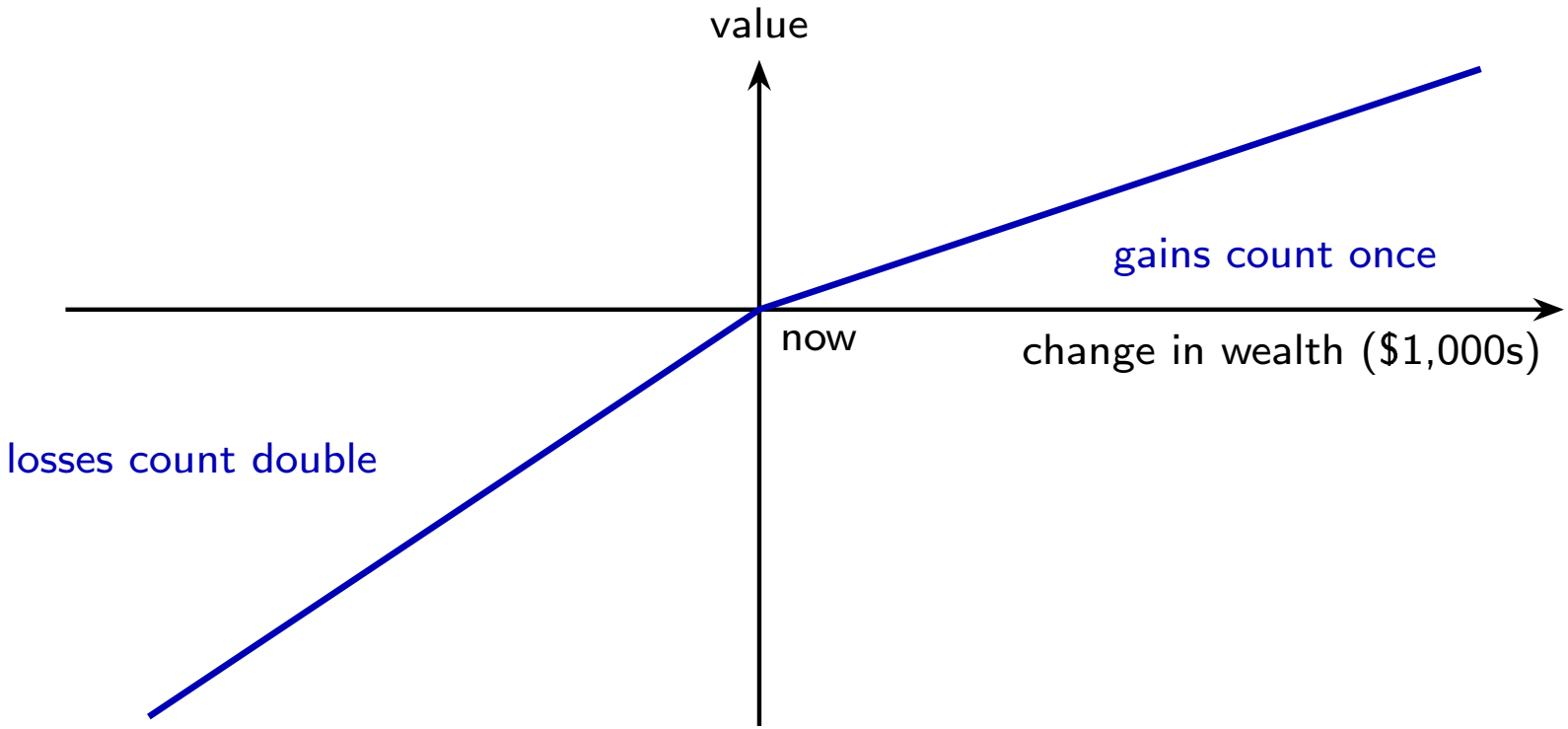
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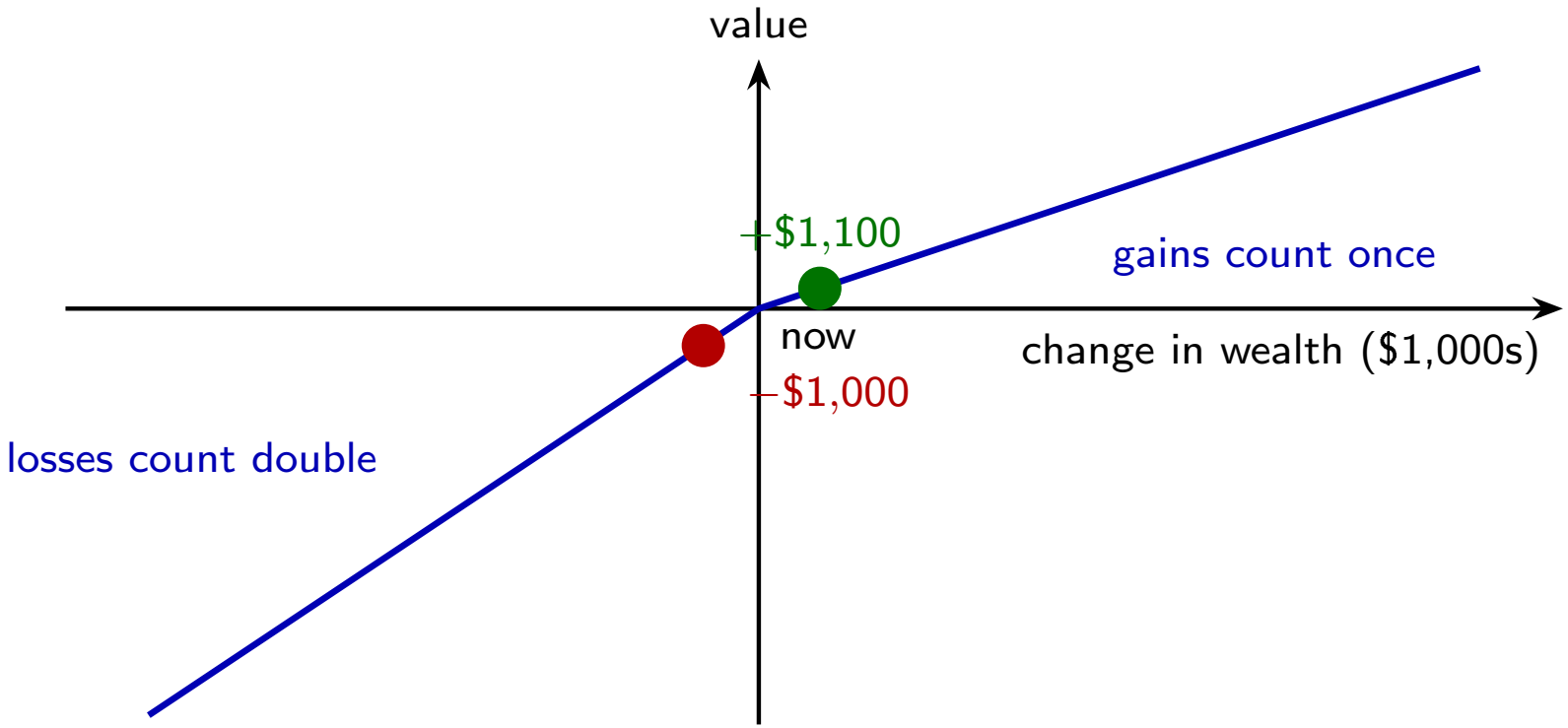
- Rabin's explanation is *loss aversion*: people are much more averse to losses relative to the status quo than they are attracted by gains.
- On this view, utilities depend on changes in wealth, not on absolute levels.

Loss Aversion at Work



A loss-averse agent values changes from where they are now.
Say losses count twice as much as gains.

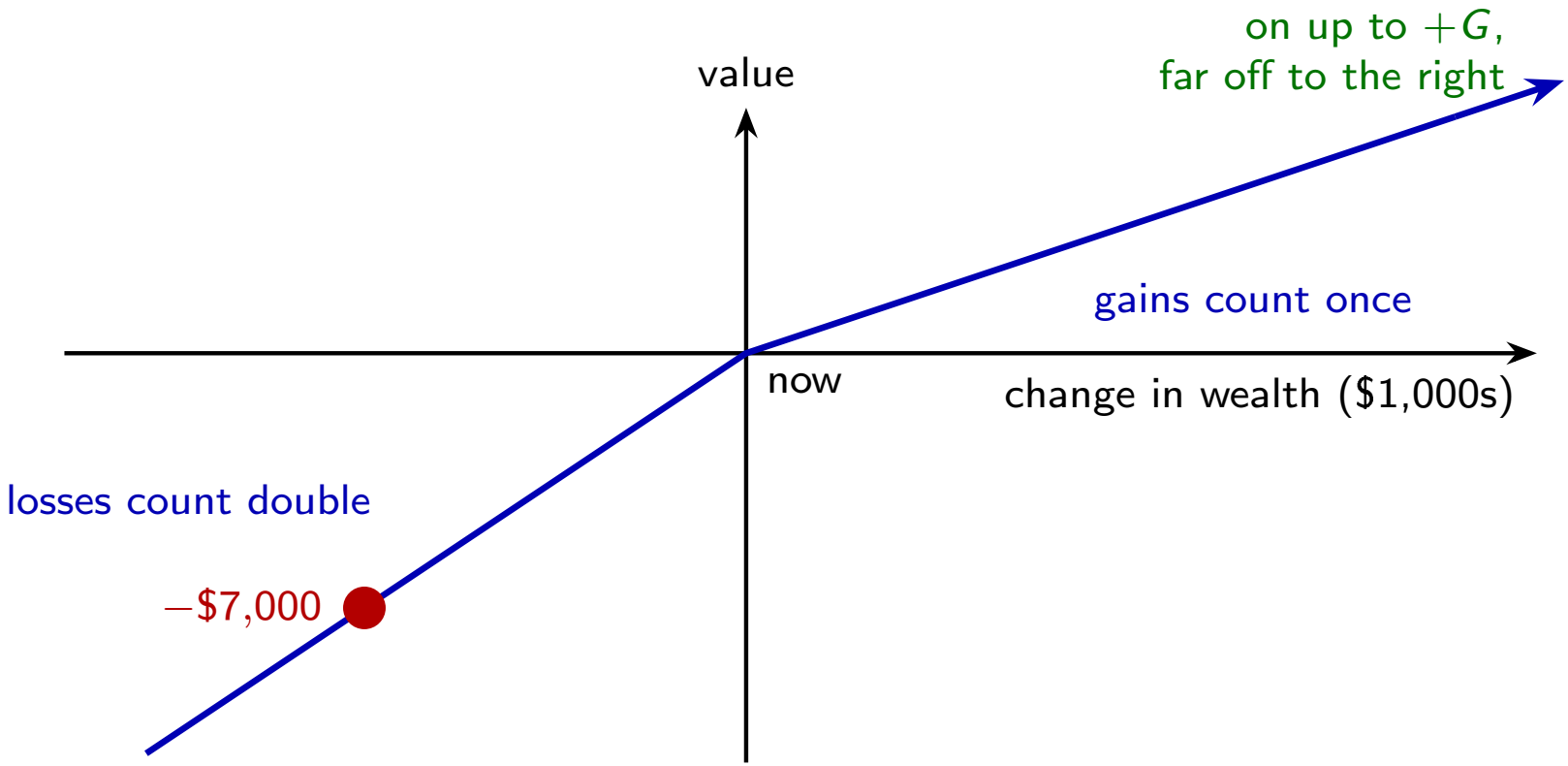
Loss Aversion at Work



Lose \$1,000 / gain \$1,100: $\frac{1}{2}(-2{,}000) + \frac{1}{2}(1{,}100) = -450$, so the agent turns it down.

The kink is always at the current wealth level,
so the agent turns this gamble down at every wealth level.

Loss Aversion at Work



Lose \$7,000 / gain G: $\frac{1}{2}(-14{,}000) + \frac{1}{2}G$.

That's positive whenever G is more than \$14,000, so the agent takes it.

References

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